**18.781 Problem Set 8**

Due Wednesday April 17 in class.

**1.** Prove that for any positive integer $m$, $\langle\overline{2m}\rangle = m + \sqrt{m^2+1}$.

**2.** Write down *all* the quadratic irrationals whose continued fraction is periodic of period 1. (Problem 1 writes down some of them.)

**3.** Write down *all* the quadratic irrationals whose continued fraction is periodic of period 2.

**4.** Suppose that $a$ and $b$ are strictly positive integers. Explain why $a^2b^2 + 4ab$ is *not* a perfect square. (It's OK to say "this follows from Problem 3" if you add a couple of additional words of explanation. But it's also possible to solve this problem independently.)

**5.** Calculate the continued fraction expansion of $\sqrt{61}$.

Summary of the method from the text and class for calculating the expansion of $(m_0 + \sqrt{d})/q_0$: make a table with rows numbered $i = 0, 1, 2, \ldots$, and four columns of data: $m_i$, $q_i$, $\xi_i = (m_i + \sqrt{d})/q_i$, and $a_i = [\xi_i]$. Calculate row $i+1$ from row $i$ by the formulas

$$m_{i+1} = q_i a_i - m_i, \qquad q_{i+1} = (d - m_{i+1}^2)/q_i.$$

This works as long as $m_0$ is an integer, $d$ is a positive integer non-square, and $q_0$ is a positive divisor of $d - m_0^2$. Then (for all $i \geq 1$) $d - m_i^2 > 0$, and $q_i$ is a positive divisor of $d - m_i^2$.