

18.781 Problem Set 1

Due Monday, September 12 in class.

1(a). Write 2005 in base 17.

1(b). What was the most recent year divisible by 17?

1(c). What was the most recent year divisible by 289?

2. Division with remainder works for dividing by *any* positive integer, including 1. Explain why this does not lead to a “base 1” representation of any positive integer as a string of zeros.

3. A *Pythagorean triple* consists of three integers x , y , and z such that $x^2 + y^2 = z^2$. The triple is called *primitive* if x , y , and z have no common prime factor. (So $(3, 4, 5)$ is primitive, but $(18, 24, 30)$ is not.) Euclid showed that if (x, y, z) is a primitive Pythagorean triple, then there are integers u and v so that

$$\{x, y\} = \{u^2 - v^2, 2uv\}.$$

Find u and v for the Pythagorean triple $(65, 72, 97)$. (It’s possible to proceed by pure guesswork; or you can do some thinking to direct your guessing; or you can analyze section 1.7 of the text to see how to write down u and v immediately.)

4(a). Write down the equation of the tangent line at the point $(5, 11)$ to the curve $y^2 = x^3 - 4$.

4(b). Find a rational solution of the equation $y^2 = x^3 - 4$ *other than* $(5, \pm 11)$. (One possible hint would be to look at the problems in the text for section 1.7, but I gave another hint in class Wednesday.)