Representations of Lie Groups 18.758

The goal of this course is the classification of the (possibly infinite-dimensional) irreducible representations of reductive Lie groups. The result is due to Langlands. For him it was a piece of technical evidence for the "tissue of surmise and hypothesis" (can you find the source of that quote?) constituting the Langlands program. For me the real groups are interesting in themselves, but I am always hopeful that the detailed knowledge we can acquire about them may shed a little light on Langlands' ideas.

The central idea of the classification is to relate the representations of a reductive group G to representations of "Levi subgroups" L of G. (A Levi subgroup is the centralizer in G of a semisimple element of the Lie algebra. The smallest Levi subgroups are the Cartan subgroups of G, and the largest is G itself.) To go from G to L I will use Lie algebra cohomology with respect to nilpotent Lie algebras, a generalization of the notion of "highest weight" in the finite-dimensional theory. (Langlands used instead Harish-Chandra's analysis of the differential equations satisfied by matrix coefficients.) To go from L to G I will use Zuckerman's notion of "cohomological induction," which generalizes the construction of Verma modules. (Langlands used results of Harish-Chandra that could be described by the same sentence, but are very different in detail.)

In order to be a little more concrete, I'll offer here a statement of a special case of the Langlands classification. In order to maintain the light-hearted introductory tone, I won't define all the words.

Theorem 1 (Zhelobenko) *Suppose G is a complex connected reductive Lie group, and H is a Cartan subgroup of G. Write $W(G,H) = N_G(H)/H$ for the Weyl group of H in G. Then the set of (infinitesimal equivalence classes) of irreducible admissible representations of G is in one-to-one correspondence with the set of $W(G,H)$ orbits on the set $\widehat{H}$ of one-dimensional characters of H.*

Zhelobenko's result is in his book Гармоническийанализ на полупростых комплексных группах Ли, (*Harmonic analysis on semisimple complex Lie groups*). (I never read it. One reviewer, a stern-hearted individual famous for once failing an entire calculus class, said, "The main disadvantage of the book is that it is very difficult to read.")

The most fundamental open problem in this direction is identifying the unitary representations of G in Langlands' list. I will say as much as I can about that problem as we go along. There is now a piece of software available that understands the Langlands classification better than I do, and I hope to have the software give a guest lecture or two:

`http://atlas.math.umd.edu/software/`

The most important background for this course is the representation theory of compact Lie groups. A nice reference is Chapters IV and V of Knapp's book *Lie Groups Beyond an Introduction*, but there are many others. Necessary structure theory for Lie groups and Lie algebras will be explained (often without proofs) as needed.

There will be no required text. There is a complete proof of the Langlands classification in my (out of print) text *Representations of Real Reductive Lie Groups.* I will follow more closely my book with Knapp *Cohomological Induction and Unitary Representations*, but that does not carry out the classification. You shouldn't need to look at either of these to follow the course.

Time: MWF 2:00-3:00

Room: 2-255

David Vogan (dav@math.mit.edu, x3-4991, room 2-281)