

March 31, 2011

18.01 Problem Set 8
Due Wednesday, April 6, in recitation

Collaboration and discussion of problem sets is a good idea; you must write up your answers on your own, and you must answer question 0 of Part II.

Part I: 10 points

Notation for homework problems: “2.4/13” means Problem 13 at the end of section 2.4 in Simmons. “1A-3” means Exercise 1A-3 in Section E (Exercises) of the Supplementary Notes.

1. 4B-1ach, 2h, 5, 7; 4C-1abc, 2.
2. 4D-5, 6; 4D'-1, 4;
3. 3G-1bc, 3.

Part II: 15 points

0. Write the names of all the people you consulted or with whom you collaborated and the resources you used, beyond the course text and notes and your instructors; or say “none” or “no consultation.”
- 1) The *bell-shaped curve* is the graph of the function

$$f(t) = \frac{1}{\sqrt{2\pi}} e^{-t^2/2}.$$

The total area under this curve turns out to be 1 (this is a hard fact that we'll see later in the semester). The area under the curve between 0 and x turns out to be the probability that a (normally distributed random variable) is at most x standard deviations above the mean. This function of x is very important in probability and statistics.

- a) Sketch the graph of f .
 - b) Use Simpson's rule with two subintervals to estimate $\int_0^1 f(t)dt$; that is, the probability that a normal random variable is between the mean and one standard deviation above it. You can use a calculator to compute things like $\sqrt{2\pi}$ and $e^{-1/8}$ that come up, but keep at least six decimal places of accuracy.
 - c) The fourth derivative of f is between 1.2 and -0.5 on the interval from 0 to 1. According to the discussion on page 373 of Simmons, how big might the error in this calculation be?
 - d) Find a source on the web, or in a table in a book, that tells you the exact answer to (b) to at least six decimal places. (You should record here both the answer and your source for it.) What was the actual error in your calculation in (b)?
- 2a) Sketch the graph of the function $y = \sin^2 x$ over the interval $0 \leq x \leq \pi$. What does the picture suggest about the average value of $\sin^2 x$ on this interval?
 - b) Calculate the average value of $\sin^2 x$ over the interval $0 \leq x \leq \pi$. (One way to find an antiderivative of $\sin^2 x$ is using the formula

$$\sin^2 x = (1 - \cos(2x))/2.$$

- 3) There's an electrical engineering problem with Example 1 in section AV of the notes. What is it?