Problem 1.

(a) Solve the initial value problem \( u_t = 3u_x, \ u(x,0) = 1/(1 + x^2) \), on the interval \([-10, 10]\) using an upwind scheme with space step size \( \Delta x = 0.1 \). Decide on an appropriate time step size, and graph your solution at times \( t = 0.5, 1, 1.5 \). Discuss what you observe.

(b) Use the Lax-Wendroff scheme to solve the initial value problem. Discuss the accuracy of your solution in comparison with the upwind scheme.

(c) For what choices of step size \( \Delta t \) and \( \Delta x \) is the Lax-Wendroff scheme stable? Consider both the CFL condition and von Neumann stability analysis.

Problem 2.

(a) Let \( \beta > 0 \). Design a finite difference scheme for approximating the solution to the initial-boundary value problem

\[
\begin{align*}
&u_{tt} + \beta u_t = c^2 u_{xx}, \quad u(0,t) = u(1,t) = 0, \quad u(x,0) = f(x), \quad u_t(x,0) = g(x),
\end{align*}
\]

for the damped wave equation on the interval \( 0 \leq x \leq 1 \).

(b) Discuss the stability of your scheme. What choice of step sizes will ensure stability?

(c) Test your scheme with \( c = 1, \ \beta = 1 \), using the initial data \( f(x) = e^{-(x-0.7)^2}, \ g(x) = 0 \).