18.303 Problem Set 6

Due Thursday, 20 April 2018

Problem 1.

(a) Solve the initial value problem $u_t = 3u_x$, $u(x, 0) = 1/(1 + x^2)$, on the interval $[-10, 10]$ using an upwind scheme with space step size $\Delta x = 0.1$. Decide on an appropriate time step size, and graph your solution at times $t = 0.5, 1, 1.5$. Discuss what you observe.

(b) Use the Lax-Wendroff scheme to solve the initial value problem. Discuss the accuracy of your solution in comparison with the upwind scheme.

(c) For what choices of step size Δt and Δx is the Lax-Wendroff scheme stable? Consider both the CFL condition and von Neumann stability analysis.

Problem 2.

(a) Let $\beta > 0$. Design a finite difference scheme for approximating the solution to the initial-boundary value problem

$$u_{tt} + \beta u_t = c^2 u_{xx}, \quad u(0,t) = u(1,t) = 0, \quad u(x,0) = f(x), \quad u_t(x,0) = g(x),$$

for the damped wave equation on the interval $0 \leq x \leq 1$.

(b) Discuss the stability of your scheme. What choice of step sizes will ensure stability?

(c) Test your scheme with $c = 1$, $\beta = 1$, using the initial data $f(x) = e^{-(x-0.7)^2}$, $g(x) = 0$.