# 18.303 Problem Set 5

Due Thursday, 12 April 2018

## Problem 1.

(a) Explain how to adapt the finite-difference method for the Poisson equation with homogeneous Dirichlet boundary conditions in 2D discussed in class, to a mixed boundary value problem on a rectangle with inhomogeneous Neumann conditions.

(b) Apply your method to the problem:

$$\Delta u = 0 \quad \text{in} \quad (0,1) \times (0,1),$$

with boundary conditions:

$$u(x,0) = 0, \quad u(x,1) = 0, \quad \frac{\partial u}{\partial x}(0,y) = y(1-y), \quad \text{and} \quad u(1,y) = 0,$$

using mesh sizes $\Delta x = \Delta y = 0.1, 0.01$ and $0.001$. Compare you answers.

(c) Solve the boundary value problem via separation of variables, and compare the value of the solution and the numeral approximations at the center of the square.

## Problem 2.

(a) Design an explicit numerical scheme for approximating the solution to the initial-boundary value problem

$$\frac{\partial u}{\partial t}(x,t) = \gamma \frac{\partial^2 u}{\partial x^2}(x,t) + s(x), \quad u(0,t) = u(1,t) = 0, \quad u(x,0) = f(x), \quad 0 \le x \le 1,\ t > 0,$$

for the heat equation with a source term $s(x)$.

(b) Test your scheme when

$$\gamma = \frac{1}{6}, \quad s(x) = x(1-x)(10-22x), \quad f(x) = \begin{cases} 2|x - \frac{1}{6}| - \frac{1}{3}, & 0 \le x \le \frac{1}{3}, \\ 0, & \frac{1}{3} \le x \le \frac{2}{3}, \\ \frac{1}{2} - 3|x - \frac{5}{6}|, & \frac{2}{3} \le x \le 1, \end{cases}$$

using space step sizes $\Delta x = 0.1$ and $0.05$, and a suitably chosen time step $\Delta t$. Are your two numerical solutions close?

(c) What is the long-term behavior of the solution? Can you find a formula for its eventual profile?

(d) Design and implement an implicit scheme for the same problem. Does this affect the behavior of your numerical solution? What are the advantages of the implicit scheme?