We defined the limit of a sequence as follows.

**Definition 1.** The number $L$ is the limit of a sequence \( \{a_n\} \), if for any natural number $m$ there exists a number $N_m$ such that $|L - a_n| \leq \frac{1}{m}$ holds for all $n \geq N_m$.

It is equivalent to the definition following definition given in page 35 of the textbook.

**Definition 2.** The number $L$ is the limit of a sequence \( \{a_n\} \), if for any positive number $\epsilon > 0$ there exists a number $\bar{N}_\epsilon$ such that $|L - a_n| \leq \epsilon$ holds for all $n \geq \bar{N}_\epsilon$.

One can observe that $\epsilon$ in definition 2 plays the role of $\frac{1}{m}$ in definition 1.

When we show a sequence is unbounded, we will frequently use the following Theorem.

**Theorem 3.** Given any number $N$, there exists a large natural number $n$ such that $n \geq N$ holds.

Let us consider an example.

**Problem 4.** Show that the sequence \( \{\sqrt{n}\}_{n \geq 0} \) is not bounded above.

**Proof.** Assume that \( \{\sqrt{n}\}_{n \geq 0} \) is bounded above. Then, by definition of bounded sequences, there exists a number $B$ such that $\sqrt{n} \leq B$ holds for all $n \geq 0$. Namely, $n \geq B^2$ holds for all $n \geq 0$. This contradicts the above theorem. Therefore, \( \{\sqrt{n}\}_{n \geq 0} \) is unbounded. \[\square\]