1. Find the gcd of the pair of integers (399,119).

2. Find all possible integer solutions (x, y) to the equation

$$399x + 119y = \gcd(399, 119).$$

3. Does the congruence

$$119x \equiv 14 \pmod{399}$$

have a solution? If not, why not. If so, provide at least one solution mod 399.

4. Compute $\phi(225)$.

5. Do there exist natural numbers n and m such that $\phi(mn) \neq \phi(n)\phi(m)$? Explain why or give a counterexample.

6. Find the smallest integer N such that $\phi(n) \geq 5$ for all $n \geq N$.

7. Find TWO solutions n, where n is a positive integer, to the equation

$$\phi(n) = n/3$$

8. In this question, you ONLY need to determine whether or not the system of equations has at least one solution. If yes, mark "Y", if no, mark "N."

   (a) $x \equiv 1 \pmod{6}$
   $x \equiv -1 \pmod{18}$

   (b) $2x \equiv 1 \pmod{1234567891011}$

   (c) $x \equiv 3 \pmod{29}$
   $x \equiv 5 \pmod{47}$

   (d) $x^{22} \equiv 1 \pmod{23}$

   (e) $x^2 \equiv 4 \pmod{7}$

   (f) $x^2 \equiv 17 \pmod{63}$

   (g) $x^3 + x + 57 \equiv 0 \pmod{125}$

9. In this question, you ONLY need to determine the NUMBER of solutions mod m for each given modulus m. You do not need to determine the actual solutions.

   (a) $14x \equiv 18 \pmod{22}$

(b) $5x \equiv 2718281828 \pmod{3141592653}$

(c) $x \equiv 7 \pmod{33}$
$x \equiv 8 \pmod{21}$

(d) $x^2 \equiv 22 \pmod{5}$

(e) $x^8 \equiv 1 \pmod{20}$

(f) $x^6 + 5x^4 + 2x + 1 \equiv 0 \pmod{243}$

10. Find all primitive elements in $(\mathbb{Z}/11\mathbb{Z})^\times$

11. Determine the number of primitive elements in $(\mathbb{Z}/m\mathbb{Z})^\times$ for the following moduli m:

(a) m=23

(b) m=49

(c) m=27

(d) m=132

12. In the integers mod 17, 3 is a primitive root. Use the following table of indices to solve the subsequent equations for all x mod 17. (That is, $i(a)$ denotes the power of 3 needed to give the residue a mod 17)

a (mod 17)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
i(a) (mod 16)	16	14	1	12	5	15	11	10	2	3	7	13	4	9	6	8

(a) $x^4 \equiv 1 \pmod{17}$

(b) $7x^{11} + 5 \equiv 0 \pmod{17}$

13. Show that in $(\mathbb{Z}/p\mathbb{Z})^\times$, the number of quadratic residues mod p is always equal to the number of quadratic non-residues mod p.

14. Find the number of reduced residues a mod m such that $a^{m-1} \equiv 1 \pmod{m}$

15. Show that $x^8 \equiv 16 \pmod{p}$ always has a solution.