

**Fall 2013, Course 18.735, Topics in algebra:  
Canonical bases and representation categories**

TTh, 1-2:30, Room 4-146

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Basic numeric invariants studied in representation theory include: the number of irreducible representations in a given class, their characters or dimensions, multiplicities in the Jordan-Hoelder series in specific modules etc. In many cases these problems are solved by identifying the Grothendieck group of the category of representations with the space of representations of another ring, so that the classes of irreducible representations form the *canonical basis* in the space. This usually involves establishing a structure (grading) on the representation category and proving its properties (e.g. Koszul property) connected to deep results in algebraic geometry.

The first pioneering step in this direction was taken by Kazhdan and Lusztig in 1970's when they proposed a famous conjecture describing the characters of highest weight modules over a semi-simple Lie algebra. The course will start by a presentation of that conjecture and its proof (modulo some results from algebraic geometry). We will then proceed to generalizations to other settings such as affine Lie algebras, finite  $W$ -algebras and rational Cherednik algebras. Time permitting we will discuss the relation of this theory to wall crossing.

**Prerequisites.** Semi-simple Lie algebras over  $\mathbb{C}$  and their finite dimensional representations, basic algebraic geometry.

**Course material** will be posted at:  
[http : //math.mit.edu/ ~ bezrukav/18\\_735.html](http://math.mit.edu/~bezrukav/18_735.html)

**Office hours.** M 3-4; you are also welcome to stop by my office any time, send me an e-mail in advance if you want to make sure I am available.

**Final grade.** Registered students must meet the instructor at least twice during the semester to discuss the subject matter of the course. Their understanding of the material will be evaluated based on those conversations and the final grade will be determined accordingly.