18.325: Topics in Applied Mathematics: Random Walks and Diffusion

Spring 2001 – Massachusetts Institute of Technology – *Revised Feb 13, 2001*

Time & Place: Tuesdays & Thursdays 11:00am–12:30pm, Room 2-105.
Instructor: Prof. Martin Z. Bazant, bazant@math.mit.edu, 253-1713.
Office Hours: Mondays 2-3pm, Wednesdays 3-4pm, 2-363B.
Three Problem Sets: Due on Tuesdays... February 27, March 20, May 1. The homeworks will be self-graded, as soon as the solutions become available. Instructor will do some quality control.
Lecture Summaries and Homework Solutions: Each student will write detailed summaries (ideally in LaTeX) of one or two lectures and/or solutions for some of the homework problems.
One Final Project: The topic must be approved by Tuesday April 24. A brief presentation will be given on May 15 or May 17. A written report is due by Monday May 21. (No final exam!)
Grading: Grading will be based on the problem sets (30%), lecture summaries and/or problem solutions (20%), and the final project presentation (20%) and report (30%).
Required Books: None. The course will be based on original lectures and handouts. A review of probability from Chap. 1 of Bouchaud and Potters (below) is available at MIT CopyTech.
Recommended Books: Barry Hughes, *Random Walks and Random Environments*, Vol. 1 (Oxford, 1996); J. Crank, *Mathematics of Diffusion* (Oxford, second ed., 1975); D. Stauffer and A. Aharony, *Introduction to Percolation Theory* (Taylor & Francis, 1992); J.-P. Bouchaud and M. Potters, *Theory of Financial Risks* (Cambridge, 2000); H. C. Berg, *Random Walks in Biology* (Princeton, 1983); N. G. van Kampen, *Stochastic Processes in Physics and Chemistry* (Elsevier, 1992); Barry Hughes, *Random Walks and Random Environments*, Vol. 2 (Oxford, 1996).

COURSE OUTLINE

1. **Random Walks and Diffusion**
   (a) Bernoulli, Polyá, Pearson, and Rayleigh walks
   (b) Bachelier-Kolmogorov equation, moment expansion, Fokker-Planck equation, Einstein relation
   (c) Green functions, intermediate asymptotics, regular and singular perturbations
   (d) Fourier/Laplace analysis, Central Limit Theorem(s), saddle point method
   (e) Extreme events, order statistics, extent of the central region, fat tails
   (f) Lévy flights, anomalous (super)diffusion
   (g) Polyá's theorem, return, first passage
   (h) Adsorption, capture, surface growth
   (i) Itô calculus, Langevin/Fokker-Planck "calculus"
   (j) Options pricing and hedging: Black-Scholes and beyond
   (k) Montroll-Weiss continuous-time walks, master equations, anomalous (sub)diffusion
   (l) Queueing theory and Langmuir adsorption

2. **Random Environments**
   (a) Percolation, cluster statistics, spanning probability
   (b) Renormalization groups and limit theorems
   (c) Random walks in random environments

3. **Interacting Random Walkers**
   (a) Electrolytes, Debye screening, boundary layers
   (b) Reaction-diffusion fronts