

Math 53 Homework 11

Due Tuesday 11/15/11 in section

(The problems in parentheses are for extra practice and optional. Only turn in the underlined problems.)

Monday 11/7: Curl and divergence; vector forms of Green's theorem

- **Read:** section 16.5.
- **Work:** 16.5: 1, (5), (7), 9, 11, (12), 15, (16), (21), 25, (26), (32), 33, 34, 36*, 37, (38).

Problems 1–3 below.

* Remark: 16.5 # 36 shows that, if a harmonic function f on a domain D is zero everywhere at its boundary curve C , then $f = 0$ everywhere in D . A consequence is that, if f_1 and f_2 are two harmonic functions in D such that $f_1 = f_2$ at every point of C , then $f_1 = f_2$ everywhere in D (by applying the previous result to the harmonic function $f_1 - f_2$). This uniqueness property has important applications in mathematical analysis.

Wednesday 11/9: Surface area

- **Read:** section 16.6.
- **Work:** 16.6: (3), 13, (18), 23, (24), 25, (32), (37), 41, 43, (45).

Friday 11/11: no class (Veterans' Day)

Problem 1. This problem shows how the value of the integral $I_n = \int_0^{2\pi} \cos^{2n} \theta \, d\theta$ can be determined for all n using Green's theorem (the conventional method is via integration by parts).

a) Prove that $I_n = \frac{2n-1}{2n} I_{n-1}$ by writing Green's theorem for the line integral $\oint_C x^{2n-1} dy$, where C is the unit circle $x^2 + y^2 = 1$ counterclockwise, and by evaluating each of the integrals separately until it looks like either I_n or I_{n-1} .

b) What is the value of I_0 ? Using the result of (a), find expressions for I_1 , I_2 , I_3 (don't simplify fractions or calculate products), then give a general formula for I_n .

Problem 2.

a) Let C be the unit circle, oriented counterclockwise, and consider the vector field $\vec{F} = x^2 \hat{i} + xy \hat{j}$. Describe geometrically and/or sketch the vector field \vec{F} (see also 16.2 # 32(b) assigned on HW 10). Which portions of C contribute positively to the flux $\int_C \vec{F} \cdot \hat{n} \, ds$? Which portions contribute negatively?

- b) Find the flux of \vec{F} through C by directly evaluating the line integral $\int_C \vec{F} \cdot \hat{n} \, ds$. Explain your answer using (a).
- c) Find the flux of \vec{F} through C by using the second vector form (“normal form”) of Green’s theorem.

Problem 3.

Find the flux of the vector field $\vec{F} = \frac{x\hat{i} + y\hat{j}}{x^2 + y^2}$ outwards through any circle centered at $(1, 0)$ of radius $a \neq 1$. Consider the cases $a > 1$ and $a < 1$ separately, and use Green’s theorem (carefully!). Explain your answers with diagrams.