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**Adin, Ron M.** (IL-BILN-CS)**;**
**Postnikov, Alexander [Postnikov, A. E.]** (1-MIT-DAM)**;**
**Roichman, Yuval** (IL-BILN-CS)

**Hecke algebra actions on the coinvariant algebra. (English. English summary)**

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Let $P_n = F[x_1, \cdots, x_n]$ for $F$ a field of characteristic zero. The symmetric group $S_n$ acts on polynomials in $P_n$ by permuting the variables and leaves invariant the ideal $I_n$ generated by the symmetric polynomials without constant term. The algebra $P_n/I_n$ is called the coinvariant algebra of type $A$.

The authors define two actions of the Hecke algebra of type $A$ on $P_n$, both of which deform in different directions the standard $S_n$ action mentioned above. One uses divided difference operators which have appeared in the work of A. Lascoux and M.-P. Schutzenberger [Funktsional. Anal. i Prilozhen. 21 (1987), no. 4, 77–78; MR 89d:16046], F. Hirzebruch [ Topological methods in algebraic geometry, English translation, Third enlarged edition, Springer-Verlag New York, Inc., New York, 1966; MR 34#2573] and G. Duchamp et al. [Publ. Res. Inst. Math. Sci. 31 (1995), no. 2, 179–201; MR 96h:05208]. The other is a "randomized action" related to an action defined by M. Jimbo [Lett. Math. Phys. 10 (1985), no. 1, 63–69; MR 86k:17008] in the quantum group context. Both actions leave $I_n$ invariant, and so the actions both lift to the coinvariant algebra, where they are shown to define equivalent representations of the $k$th homogeneous components of $P_n/I_n$.

The main result is a nice character formula for both of these representations (on the Coxeter element of a Young subgroup), as a sum of weights indexed by elements of a certain length in $S_n$. It uses (among other things) a basis of Schubert polynomials for the coinvariant algebra and a variation of Monk's formula for certain products of Schubert polynomials due to I. G. Macdonald [in Surveys in combinatorics, 1991 (Guildford, 1991), 73–99, Cambridge Univ. Press, Cambridge, 1991; MR 93d:05159].

The result is motivated by a desire to find a $q$-analogue to a result of Y. Roichman [Discrete Math. 217 (2000), no. 1-3, 353–365; MR 2001g:05104], which gives a similar weight formula for the characters of $S_n$ on the homogeneous components of the coinvariant algebra. However, it is also related to earlier work of the same author on Kazhdan-Lusztig characters [Adv. Math. 129 (1997), no. 1, 25–45;

MR 98m:20020], which gave expressions as sums of the same weights but with different indexing set. *Andrew R. Francis* (Richmond)

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authored with R. M. Adin and A. Postnikov), *Discrete Math.*, to appear. MR 2001g:05104