# Practice Problems for Exam # 3 on 12/02/2004

These practice problems will give you an idea of what the actual exam is like. The problems in the actual exam may not be parallel or analogous to the practice problems.

1. Find the number of spanning trees of the complete graph $K_5$ with one edge removed: $G = K_5 \setminus e$.

2. Let $G$ be a regular graph. Suppose that the eigenvalues of the adjacency matrix of $G$ are $-2, -2, -2, -2, 1, 1, 1, 1, 4$.

   (a) Find the degree, the number of vertices, and the number of edges of the graph $G$.

   (b) Find the number of spanning trees of $G$.

3. Find eigenvalues of the adjacency matrix for the complete bipartite graph $K_{3,3}$.

4. Let $C_n$ be the graph that is the cycle of length $n$. Suppose that two adjacent vertices are colored with two distinct colors from $\{1, \ldots, k\}$. In how many way can this partial coloring be extended to a proper coloring of $C_n$ with the colors in $\{1, \ldots, k\}$.

5. Find the number of sequences $(a_1, a_2, a_3)$, $a_i \in \{1, 2, 3\}$ such that the 3 sets $\{1, \ldots, a_1\}$, $\{1, \ldots, a_2\}$, $\{1, \ldots, a_3\}$ have a system of distinct representatives.