Last Time: any monomial \( x^\lambda = x_1^{\lambda_1} \cdots x_n^{\lambda_n} \) s.t.

1. \( 0 \leq \lambda_i \leq n-i \) for \( i = 1, \ldots, n \)
2. \( \lambda_1 \geq \cdots \geq \lambda_n \)

is a Certain Schubert Polynomial.

"Special" divided difference

\[ \partial_i \ x^\lambda \rightarrow x_1^{\lambda_1} \cdots x_i^{\lambda_i-1} \cdots x_n^{\lambda_n} \]

where \( \lambda_i = \lambda_{i+1} - 1 \)

Questions: what are corresponding weSn.
Lehmer code of perm. \( w \in S_n \)

\[
\text{code}(w) = (c_1, \ldots, c_n) \text{ s.t.}
\]

where \( c_i = \# \{ j > i \mid w_j < w_i \} \)

\[
\begin{align*}
\text{Ex} & \quad w = (1 \, 5 \, 2 \, 3 \, 4) \\
\text{code} &= (0 \, 3 \, 0 \, 0 \, 0)
\end{align*}
\]

Lemma: \( w \mapsto \text{code} \) is a bijection between \( S_n \) and \( \{(c_1, \ldots, c_n) \mid 0 \leq c_i \leq n-i\} \)

Lemma: TFAE for \( w \in S_n \)

(A) code\( (w) = (c_1, \ldots, c_n) \) satisfies \( c_1 \geq \ldots \geq c_n \)

(B) \( w \) is 132-avoiding

no \((i,j,k)\) with \(ijck\) s.t. \( w_i < w_k < w_j \)
Ex \( w = 2 \ 1 \ 4 \ 5 \ 3 \)

code = 1 0 1 1 0

Def 132-avoiding permutations are called dominant permutation.

Thm For a dominant permutation \( w \in \mathfrak{S}_n \)

\[ S_w = x^{\text{code}(w)} \]
The Demazure operators $D_i$ (aka isobaric divided differences) for $i = 1, \ldots, n-1$ and $f \in \mathbb{C}[x_1, \ldots, x_n]$ def:

$$D_i : f \rightarrow \left(f - \frac{x_{i+1}}{x_i} s_i f \right) / \left(1 - \frac{x_{i+1}}{x_i} \right)$$

In other words $D_i = \partial_i (x_i f)$

Lemma: $D_i$ satisfy

1. $D_i^2 = D_i$
2. $D_i D_{i+1} + D_{i+1} D_i = D_{i+1} D_i D_{i+1}$ for $i = 1, \ldots, n-1$
3. $D_i D_j = D_j D_i$ if $|i-j| > 2$.

O-Hecke relations

Hence for $w = s_{i_1} s_{i_2} \cdots s_{i_e}$ (reduced),

$$D_w : = D_{i_1} \cdots D_{i_e}$$
**Def** Key polynomials (aka Demazure Char.)\[ \lambda = (\lambda_1, ..., \lambda_n) \] and we set
\[ \chi_{\lambda, \omega} (x_1, ..., x_n) = D_\omega (x^\lambda) \]

**Thm** If \( \omega = \omega_0 \in S_n \) then
\[ \chi_{\lambda, \omega_0} (x_1, ..., x_n) = s_\lambda (x_1, ..., x_n) \]

**Proof:** By def. \( s_\lambda = \theta_{\omega_0} (x^\lambda + \delta) \)
\[ \chi_{\lambda, \omega_0} = D_{\omega_0} (x^\lambda) \]
we need to show that
**claim** \( \forall f \in \mathbb{C}[x_1, ..., x_n] \)
\[ \omega_0 (x^\delta f) = D_{\omega_0} (f) \]