

18.100A Fall 2012: Assignment 9 due Wed. Oct. 10

**Directions:** You can collaborate, but must write up the solutions independently, i.e., thinking through the problem by yourself and expressing it in your own words. Consulting solutions to problem sets of previous semesters is not allowed. Cite by name or number any significant theorems you are using in your arguments..

**Reading: Chaps. 11.1-11.4**

11.1 continuity

11.2 definition of  $\lim_{x \rightarrow x_0} f(x)$  and basic practice with it;

11.3 limit theorems: algebraic theorems, using  $+$ ,  $\cdot$ ,  $/$  ;  
comparison theorems (squeeze, location), using inequalities .

These are analogous to the limit theorems for sequences in 5.3; the proofs are similar.

Read the names of the theorems, shut the book, and see if you can write down what they should say, based on what they say for sequences (which are a special kind of function).

11.4 to p. 163 connection between continuity and limits (omit discontinuities for now).

The material may seem to jump back and forth between different topics.

There are two equivalent definitions of continuity of  $f(x)$  at  $x_0$ : the basic one in 11.1 (using  $\epsilon$  and a hidden  $\delta$ ), and the one in 11.4 (using limits, and called Theorem 11.4A).

In between, the definition of limit has to be given, and simple theorems about limits proved. The advantage of the limit definition for continuity is that these limit theorems often give quick proofs of continuity properties. Sometimes however the basic definition of continuity gives a better way to think about a particular problem.

Continuity at  $x_0$  has to be extended in two ways, which have no analogue for sequences. One is to continuity on an interval, which can be finite or infinite. The other is to right-hand and left-hand continuity at  $x_0$ , to explain what continuity at the end-points of a finite closed interval should mean. This is done in 11.1 and 11.2.

**Problem 1.** (2) Work 11.1/4. (The exponential law is  $e^{a+b} = e^a e^b$  .)

**Problem 2.** (2) Work 11.3/3a.

(One of the two inequalities needed to apply the Squeeze Theorem is a bit harder to find than the other. Keep in mind what the variable is in the theorem. )

**Problem 3.** (2) Work 11.3/6ab.

Write the polynomial  $f(x)$  in the factored form:  $a_0 x^n \left( 1 + \frac{a_1}{a_0} \left( \frac{1}{x} \right) + \dots + \frac{a_n}{a_0} \left( \frac{1}{x} \right)^n \right)$ .

Use limit theorems.

**Problem 4.** (2: 1.5, .5) Prove (20) and (21) in Theorem 11.3C, the Limit Location Theorem for functions. Try to do this without looking back at the proof of the corresponding theorem for sequences.

**Problem 5.** (2) Work P11-3b.

Use the basic definition of continuity in 11.1 .

Given  $\epsilon > 0$ , how small must  $\delta$  be to guarantee that  $x \approx_{\delta} a \Rightarrow f(x) \approx_{\epsilon} f(a)$ ?

The answer will somehow have to involve  $1/2^k$  for some suitable value of  $k$ .