18.100A Fall 2012: Assignment 7 due Mon. Oct. 1

Directions: List collaborators; do not consult assignments from previous semesters; cite relevant Theorems or Examples.

Reading: 8.1, 8.2 (omit Abel), **8.3; 8.4** (8.4: statements only, omit proofs)
Power series: radius of convergence, endpoints, addition and multiplication.

Problem 1. (2.5: 1.5, 1) For each of the following power series, find its radius of convergence R, with proof, and examine whether it converges or diverges at the endpoints $x = \pm R$, with proof. Identify the test being used. Check your calculation of R; if you get the wrong value, the work on the endpoints will be useless.

$$a)\ \sum_1^\infty \frac{x^n}{\sqrt{2^n n}} \qquad\qquad b)\ \sum_0^\infty \frac{(-1)^n 3^{2n} x^n}{n!}$$

Problem 2. (2) Follow the directions in Problem 1 for $\displaystyle\sum_1^\infty \frac{(2n)!}{(n!)^2}\, x^n$.

Determining convergence at the endpoints is harder here; use: $\displaystyle\frac{(2n)!}{(n!)^2 2^{2n}} \sim \frac{1}{\sqrt{\pi n}}$, as $n \to \infty$. (You will be able to prove this later in the semester.) Note that verifying the hypotheses of the theorems you are using here requires some care.

Problem 3. (2) a) Work 8.1/2 b) Work 8.1/3

Problem 4. (1) Work 8.3/1

Problem 5. (2.5: .5,.5, 1.5)

a) Work Question 8.4/1.

b) Work 8.4/1a(i).

c) Work 8.4/1b.

Prove (*) by multiplying the two power series for $f(x)$ and $1/(1-x)$; the other statement is proved in the chapter and you can just cite it if you can find it, but try proving it yourself from scratch (note that the a_n can be negative, so absolute values will be needed in the argument).