

18.100A Fall 2012: Assignment 4 due Mon. Sept. 17

Directions: You can collaborate, but should list those you worked with and write up the solutions independently (i.e., not copying but thinking them through by yourself).

Consulting solutions to problem sets of previous semesters is not allowed.

When used significantly, cite theorems by name or number to justify statements or implications.

Reading: Chapter 5.2-.5; 6.2 (for statements)

Limit theorems involving inequalities: Squeeze theorem, Location theorems; subsequences; applications; cluster point definitions and examples.

Problem 1. (4) The four parts are independent of each other; the last three use the Sequence Location Theorem in a short proof, the first one deals with the $<$ vs. \leq symbols.

a) Work 5.3/1 (possible versions of the theorem when $L = 0$)

Try to decide without first consulting the theorem; you can use it later as a partial check on your answers.)

b) Prove that $\lim x_n = L \Rightarrow \lim |x_n| = |L|$.

(There are three cases, depending on the sign of L . The proof for the case $L = 0$ is different from the other two.)

c) Prove the limit location theorem in the form (15b) indirectly: assume $\lim a_n < M$ and derive a contradiction by using the sequence location theorem.

d) Work Exercise 5.3/6, by using the sequence location theorem and algebraic limit theorems from 5.1 .

Problem 2. (2) Read section 5.2 on the Squeeze Theorem, then work 5.2/3 , following the hint given there.

(Interpret the sum a_n as the total area of rectangles, find areas under the curve $y = \sqrt{x}$ that give upper and lower estimates for a_n by shifting the rectangles right or left, then apply the squeeze theorem to prove the asymptotic estimate given for a_n in the problem.)

Problem 3. (2) Work 5.4/2, using the suggestions given.

Problem 4. (2)

Section 6.2 defines what is meant by a *cluster point* of a sequence a_n .

Theorem 6.2 (Cluster Point Theorem) gives what amounts to an alternative definition that's often used instead, and shows that the two definitions are equivalent. You can skip its proof for now, but study the examples in this section.

Then apply the alternative definition given by Theorem 6.2 to the sequence in Problem 3, to prove that all the rational numbers $1/n$, $n = 1, 2, 3, \dots$, are cluster points of the sequence.

(An easier problem (for 1 point) is to just prove the statement when $n = p$, a prime number.)