18.100A Fall 2012: Assignment 3 due Fri. Sept 14.

Directions: You can collaborate, but should list those you worked with and write up the solutions independently (i.e., not copying but thinking them through by yourself).
Consulting solutions to problem sets of previous semesters is not allowed.

Reading: Mon. Chapter 3.1-6 Definition of limit, $\lim a^n$, examples using integrals.
　　　　　Wed. Chap. 4.1-.2, 4.4; 5.1 The error form for limits; examples: geometric and related sequences, Fibonacci sequence. Algebraic limit theorems.

Problem 1. (2 pts.) Prove directly from the definition of limit in Chapter 3 (i.e., without using any limit theorems you may have learned in calculus, or just feel are intuitively obvious, like $\lim 1/n = 0$) that
$$\lim_{n \to \infty} \frac{n^3 + 2n - 1}{2n^3 + n} = \frac{1}{2} .$$
(Remember that you don't have to find the exact value of N; you can simplify the inequalities as we did in class Mon. to make it easier to find some value of N that will work. If you are trying to find N by solving some polynomial equation, you are working too hard.)

Problem 2. (2 pts.) Work 3.2/5

Problem 3. (2 pts.) Prove that if $a > 1$, then $a^n/n^2 \to \infty$.
(Imitate the proof of Theorem 3.4, but use a different term in the binomial expansion.)

Problem 4. (2 pts.) Work 3.6/1b. (You will need Theorem 3.4 .)

Problem 5. (2 pts.) Work 4.2/1ab.
(This problem is a good illustration of the importance of being able to estimate integrals, rather than calculate them.)

Problem 6. (2 pts.) Work 5.1/2ab (Follow the instructions.)

Problem 7. (3 pts.) Work 4.4/1, assuming that $A/B > 1$.
Denote the limit asked for by L, and determine what L is in several steps, as follows:

a) Let $a_0 = A/B$, and a_n be the n-th term of the resulting sequence. Prove the recursion formula $a_{n+1} = \dfrac{2a_n}{a_n + 1}$.
Use the algebraic limit theorems and this recursion formula to find the two possibilities for L.
　　Then show from the recursion formula that only one is possible.
　　(It may help to experiment first with numerical values for A and B, calculating a few terms of the sequence. Remember the initial assumption about A/B.)

b) Using this value for the limit L, find the recursion formula for the corresponding error term e_n, as done in section 4.4.
　　Then prove that $\lim_{n \to \infty} e_n = 0$, thus establishing that L is indeed the limit of the sequence.