18.100A Fall 2012: Assignment 22 due Mon. Nov. 26

The rules are the same as for previous assignments.

Reading: 24.6-.7; p. 364; Notes on Open and Closed Sets *(e-mailed)*

Compact sets in $\mathbf{R}^2$; three theorems about a continuous function on a compact set S; Open sets and closed sets in $\mathbf{R}^2$. (Note that the Extremal Value Theorem (24.7B) should read "non-empty compact set").

1. (1) Work Question 24.7/2.

This uses the Extremal Value (Maximum) Theorem 24.7B, which is about a certain continuous function of two variables.

2. (3)

a) Work 24.7/1 .

b) Let S be a subset of $\mathbf{R}^2$. Prove: S is compact $\Rightarrow$ S is bounded.
(Use one of the theorems in 24.7.)

c) Let S be a subset of $\mathbf{R}^2$ and $\mathbf{a}$ a cluster point of S. Prove: S is compact $\Rightarrow$ $\mathbf{a} \epsilon S$.

3. (2) Let $\{x_n\} \to$ $\mathbf{a}$ be a convergent sequence, where the $\mathbf{x}_n$ are cluster points of a set S in $\mathbf{R}^2$. Prove that $\mathbf{a}$ is a cluster point of S.

(Note that the $\mathbf{x}_n$ are not assumed to be in S. Use one of the three equivalent definitions of cluster point in the book.)

4. (2) Let $g(\mathbf{x})$ be a continuous function on $\mathbf{R}^2$, and define

$$\mathcal{D}^+ = \{\mathbf{x} : \ g(\mathbf{x}) > 0\} \ \text{ and } \ \mathcal{D}^- = \{\mathbf{x} : \ g(\mathbf{x}) < 0\}.$$

Using and citing just the theorems in Chapter 24, p. 364, and the Notes,

a) prove the two sets are open sets, using the second definition of open set in the Notes;

b) prove the sets $\bar{\mathcal{D}}^+ = \{\mathbf{x} : \ g(\mathbf{x} \geq 0\}$ and $\bar{\mathcal{D}}^- = \{\mathbf{x} : \ g(\mathbf{x}) \leq 0\}$ are closed sets.

5. (2) For each of the 10 sets S in 25.3/1, reproduce its definition, draw a sketch, tell if S is open, closed, or neither, and indicate its boundary $\partial(S)$ somehow: on the sketch, using a function or equation, or a few words..

(Short answer: no proofs or reasons are asked for. Use the definitions of open and closed sets in the Notes.)