

18.100A Fall 2012: Assignment 2 due Mon. Sept. 10

You can collaborate, but should list those you worked with, and should write up the solutions independently (i.e., not copying but thinking them through by yourself).

Consulting solutions to problem sets of previous semesters is not allowed.

Reading: Chapter 2. Estimations, absolute values, approximations, “for n large”.

Problem 1. (1 pt.) Work 2.1/3 (Exercise 2.1/3 at the end of Chapter 2.), using the directions:

Show the statement is false, strengthen the hypotheses, and prove the amended statement.

(Read top p. 405 for “stronger statement”: here the “statements” are the hypotheses on the two sequences).

Problem 2. (2 pts.) Work 2.4/2 (Exercise 2.4/2 at the end of Chapter 2.)

(Use contraposition: not $B \Rightarrow$ not A , but formulate the two negative statements very carefully, without using the word “not”. The phrase “for at least one of the a_i ” is generally written in the mathematically equivalent form “for some a_i – in ordinary writing, “some” usually means “more than one”, but in mathematics it does not, it means “more than none”.

Problem 3. (3 pts.: 1.5, 1.5)

(a) Work 2.5/4 (Exercise 2.5/4), which uses an important idea to give a nicer proof of Theorem 2.5(i) than the one in the book, since it avoids using the infinite decimal representation of a real number.

(b) Using Theorem 2.5, prove (without using infinite decimals) that, given any real number c , there is a decreasing sequence $\{x_n\}$ of rational numbers having c as its limit, i.e., its greatest lower bound.

(You must describe how to construct the sequence $\{x_n\}$, show that carrying out the construction is actually possible, show that the sequence is decreasing, and show that c satisfies the two properties of a greatest lower bound for $\{x_n\}$:

(i) $x_n \geq c$ for all n (i.e., c is a lower bound)

(ii) $x_n \geq b$ for all $n \Rightarrow b \leq c$ (i.e., c is the *greatest* lower bound).

(Use contraposition for this last step.)

Problem 4. (2 pts) Work 2.6/1, assuming in addition $a > 1$.

(In general there are two basic ways of showing that a positive sequence x_n is say increasing, i.e., that $x_n \leq x_{n+1}$: show the ratio $x_{n+1}/x_n \geq 1$. or show the difference $x_{n+1} - x_n \geq 0$.

Here in addition you have to exhibit a suitable value of N for the “for $n \gg 1$ ” condition. N will depend upon a , of course.)

Problem 5. (2 pts.)

a) Prove that, given a small number $\epsilon > 0$, $\frac{3n}{n-2} \approx \frac{3}{\epsilon}$, for $n \gg 1$.

(You must exhibit an N which works; it will depend on ϵ .)

b) Prove that the sequence defined recursively below is monotone for n large:

$$x_{n+1} = \frac{n^2 + 15}{(n+1)(n+2)}x_n, \quad x_0 > 0.$$