18.100A Fall 2012: Assignment 14 due **Fri. Oct. 26**

Directions: You can collaborate, but must write up the solutions independently. Consulting solutions to problem sets of previous years is not allowed. Cite significant theorems.

Reading: (Mon.) 15.1-.4 Differentiation and local $\to$ global theorems.
Mean-value Theorem; geometric and analytic form, estimations. L'Hospital's rule.

Problem 1. (1) Work 15.1/2, using an interval $[0, a]$ – you can choose the $a > 0$. Prove your assertion by using theorems, not by calculating derivatives.

Problem 2. (2: 1.5, .5) Work 15.2/1ab

Problem 3. (2) Work 15.3/2b.
Equation (15) comes from cross-multiplying Cauchy's form of the mean-value theorem (Theorem 15.3). It's slightly stronger than Cauchy's form, since it doesn't require the hypotheses about the denominator not being zero.

In the proof, you won't need the full force of the mean-value theorem, only the special case called Rolle's theorem, in the slightly more general form

$$f(a) = f(b) = K \quad \Rightarrow \quad f'(c) = 0 \text{ for some } c \text{ in } (a, b).$$

This follows from the usual Rolle's theorem by applying it to $f(x) - K$.

Problem 4. (2: 1,1) L'Hospital's rule.
(a) Work 15.4/1, proving the elementary case, where the rule works after one step.
(b) Work 15.4/2, as an example where several steps are required, so the full rule is needed.

Reading: (Wed.) 16.1, 16.2 through 16.2B. Linearization and convexity.
17.1-.4 skip proofs. Taylor's theorem with derivative remainder, Taylor series.

Problem 5. (1) Work 16.1/1a. Give the sharpest upper estimate that the theorems provide (as opposed to the sharpest that exists, which might require more sophisticated metods to establish.)

Problem 6. (1) a) Work 16.2/3b

Problem 7. (1.5) Suppose $f'''(x)$ exists on an interval I, and for three points $a < b < c$ on I, we have $f(a) = f(b) = f(c) = 0$, and $f'(b) = 0$.
Prove there is a point x_0 in I such that $f'''(x_0) = 0$.

Problem 8. (2: 1,1)

a) By using Taylor's Theorem, find the Taylor series for $\ln(1 + x)$ at 0.

b) By studying the remainder term in Taylor's theorem, prove that for $|x| < 1$, the series converges to the sum $\ln(1 + x)$. In proving the remainder term goes to zero, you will need a "buffer" K: prove it for $|x| \le K < 1$, for any $0 < K < 1$.

(This gives an alternative procedure to the one in section 4.1, where the series is found by a different method and the remainder term is expressed as a definite integral.)

Problem 9. (2.5: .5,1,1) Work the three parts of Problem 16-3, which give an application of Taylor's theorem for the case n = 1 (The Linearization Error theorem) to estimating the error in linear interpolation.