

## 18.100A Fall 2012: Assignment 11 due Mon. Oct. 15

**Directions:** Same as before: write up solutions independently if you collaborate; list collaborators, don't consult solutions from previous semesters; cite theorems or examples to justify arguments which use them.

### Reading: 12.1-.2

Bolzano's Theorem, in various forms, with applications.

**Problem 1.** (3) Give a different proof of Bolzano's Theorem, based on using the Completeness Principle in the form which applies to sets, given in section 6.5.

Assume  $f(x)$  is continuous on  $[a, b]$ , and changes sign from negative to positive on  $[a, b]$ , so that  $f(a) < 0$ ,  $f(b) > 0$ .

a) Consider the set  $S = \{x \in [a, b] : f(x) < 0\}$ . Define in terms of  $S$  where a zero  $c$  of  $f(x)$  lies. If there is more than one zero, will your definition give the highest zero? (If yes, no proof needed, but some reasoning; if no, give a counterexample.)

b) Prove that  $f(c) = 0$ , by using the ideas and techniques in 6.5, and the Positivity theorem for continuous functions. (Using theorems about continuous functions should allow you to avoid having to go back to the epsilon-delta definition of continuity.)

**Problem 2.** (2) Work 12.2/2ab, using instead the equation  $x^4 - \epsilon x - 1 = 0$ .

(Switch the roles of  $\epsilon$  and  $h$  used in the book to a more commonly used notation: use  $\epsilon$  as a small coefficient in the equation, and  $h$  in the zero you are estimating:  $z = a + h$ , for a suitable constant  $a$  and a small  $h$ .)

**Problem 3.** (2)

a) Prove that  $f(x) = x - \tan x$  has an infinity of positive zeros  $x_1, x_2, \dots, x_n, \dots$ . (Use the Intersection principle.)

b) For what positive value of the parameter  $A > 0$  will  $f(x) = x - A \sin x$  have exactly 4 positive zeros (i.e., 0 is not included)?

Express this value of  $A$  in terms of the  $x_n$  of part (a).

(Use calculus, give some reasoning, formal proof not asked for; a good sketch helps.)

**Problem 4.** (3) Work Problem 12-3. (Try to do this as much by yourself as possible. Some suggestions are given below.)

Try sketching the graphs of some functions  $f(x)$  which satisfy the conditions given, and seeing where the chord lies for each of them. If you have a proposed method for finding a chord, try drawing more complicated functions, to see if your idea works on those functions also.

If this sort of experimentation doesn't show the way, consider this: a chord of length  $a$  for  $f(x)$  is a line segment that has to satisfy four conditions. What are they?

The difficulty in the problem lies in having to satisfy all four conditions simultaneously. What would the line segments look like that only satisfied three of them? Try dropping each of the four conditions in turn, and pick the trial that seems to offer a path to a solution and the possibility of proving that it will always work.