18.100A Fall 2012: Assignment 10 due Fri. Oct. 12

Directions: Same as before – you can collaborate, but write up solutions independently and list collaborators; no consulting previous semesters' solutions; cite theorems being used.

Reading: 11.4, 11.5

Discontinuity types, composition of continuous functions, Sequential Continuity theorem

Problem 1. (2) Work 11.4/3. You can present your answer purely graphically. There are several possibilities, requiring separate pictures. How do you know there are no other possibilities? Cite the relevant theorems in section 11.4.

Problem 2. (2) Work 11.4/2, changing $[a, b]$ in both of its occurrences to $[a, b)$.

(This shortens the work without sacrificing any of the ideas.)

Use a limit theorem to give a direct argument – avoid an indirect proof. If you have trouble proving "strictly increasing on $[a, b)$", you can get 1.5 points by just proving "increasing".

(Focus on the minimal statement that needs to be proved; interposing an intermediate point as a "buffer" helps, like the use of M in the proof of the ratio test 7.4A.)

The next two problems are exercises in the use of the Sequential Continuity Theorem 11.5: in working them, don't go back to the basi $\epsilon - \delta$ definition of continuity.

Problem 3. (3: 1.5 each)

a) Prove that if two functions $f(x)$ and $g(x)$ are continuous on $\mathbf{R}$ and agree on all rational points, i.e., $f(a) = g(a)$ whenever a is a rational number, then $f(x) = g(x)$ for all $x \epsilon \mathbf{R}$.

(Consider their difference.)

b) Work Problem 11-3a, to show the ruler function is discontinuous at all its "special" points.

Problem 4. (2) Work P11-2, assuming $c > 0$, $c \neq 1$.

(Hint: if it is a contant function, what must the constant value be? Start with an arbitrary x, and show $f(x)$ has that value.)