Corrections to Problems in "Intro. to Analysis" (Mattuck), printings 1-7

To determine printing, e.g., Printing 3 has 10 9 8 7 6 5 4 3 on the first left-hand page.

Printing 1: needs all the items below; Printing 2: needs only those starting 2 or 3;
Printings 3 - 7: only those starting 3. Bullets mark more significant changes or corrections.

E = Exercise, P = Problem (both at end of chapter); Q = Question (at end of section)

1• E-1.3/1: *add:* (d) $\sum_0^n \sin^2 k\pi/2$

3• E-2.1/3: *replace:* "change the hypothesis on $\{b_n\}$ *by:* "strengthen the hypotheses" (cf. p. 405, Example A.1E for the meaning of "stronger")

3• E-2.2/1b: *read:* (make the upper bound sharp)

1 E-2.4/5: *replace:* c_i *by* c_k

1 E-2.5/1: *delete second* ϵ*:* $a^n \approx b^n$

1• E-2.6/4: *replace by:* Prove $\{a_n\}$ is decreasing for $n \gg 1$, if $a_0 = 1$ and

$$\text{(a)}\quad a_{n+1} = \frac{n-5}{(n+1)(n+2)}\,a_n \qquad \text{(b)}\quad a_{n+1} = \frac{n^2+15}{(n+1)(n+2)}\,a_n$$

1• P-2-4: *replace this problem by:*
A positive sequence is defined by $a_{n+1} = \sqrt{1 + a_n^2/4}$, $0 \le a_0 < 2/\sqrt{3}$.
(a) Prove the sequence is strictly increasing.
(b) Prove the sequence is bounded above.

3 E-3.3/1d: *delete the semicolons*

1• P-3-4 *add* Prove that a convergent sequence $\{a_n\}$ is bounded.

3• P-3-5 *add* Given any c in $\mathbf{R}$, prove there is a strictly increasing sequence $\{a_n\}$ and a strictly decreasing sequence $\{b_n\}$, both of which converge to c, and such that all the a_n and b_n are (i) rational numbers; (ii) irrational numbers. (Theorem 2.5 is helpful.)

3• E-4.3/2: *replace by:* For $f(x) = \ln x$, there is an x_0 such that Newton's method fails $\iff a_0 \ge x_0$. Show this (i) geometrically from the graph; (ii) analytically, from (10).

1• E-4.4/1: *replace the last line by:*
Guess what its limit L is (try an example; cf. (15), 4.4). Then by finding the recursion formula for the error term e_n, prove that the sequence converges to L
(a) if $A > B$; (b) if $A < B$.

1• E-4.4/3: *replace (b) and (c) by:*
(b) Show that the limit is in general not 1/2 by proving that
(i) $a_0 < 1/2 \Rightarrow \lim a_n = 0$; (ii) $a_0 > 1/2 \Rightarrow \lim a_n = \infty$.

1• P-4-2b: *add:*
Use the estimations $|1 - \cos x| \le x^2/2$ and $|\sin x| \le |x|$, valid for all x.

3 Ans. to Q4.3/2 (p.60): *read:* 1024

1• E-5.3/4a: *add:* (Use Problem 3-4.) *(see above on this list)*

3• E-5.4/1 *Add two preliminary warm-up exercises:*
a) Prove the theorem if $k = 2$, and the two subsequences are the sequence of odd terms a_{2i+1}, and the sequence of even terms a_{2i}.
b) Prove it in general if $k = 2$.
c) Prove it for any $k \ge 2$.

1 E-5.4/2: *add:* (Use Exercise 3.4/4.)

3• P-5-1(a): *replace the first line of the "proof" by:*
Let $\sqrt{a_n} \to M$. Then by the Product Theorem for limits, $a_n \to M^2$, so that

3 E-6.1/1a: *change* c_n *to* a_n

3• E-6.1/1b: *add:* Assume $b_n - a_n \to 0$.

add at end: to the limit L given in the Nested Intervals Theorem.
1• E-6.2/1: *make two exercises (a) and (b), and clarify the grammar:*
Find the cluster points of: (a) $\{\sin(\frac{n+1}{n}\frac{\pi}{2})\}$ (b) $\{\sin(n+\frac{1}{n})\frac{\pi}{2}\}$.
For each cluster point, find a subsequence converging to it.
1• E-6.2/2: *replace by:* A sequence $\{x_n\}$ uses only finitely many numbers $a_1, \ldots, a_k$; i.e., for all n, $x_n = a_i$ for some i (where i depends on n.) Prove $\{x_n\}$ has a cluster point.
3• E-6.2/3 *read:* Find the cluster points of the sequence $\{\nu(n)\}$ of Problem 5-4.
3• E-6.3/2 *add:* Prove the Bolzano-Weierstrass Theorem without using the Cluster Point Theorem (show you can pick an x_{n_i} $[a_i, b_i]$).
3 E-6.5/4: *read:* non-empty bounded subsets
3• Q-8.2/2 (p.118) *the series is not Abel-summable; replace by:* Show the Abel sum of $0 + 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots$ is the same as its ordinary sum (cf. 4.2).
3• P-8-2 *add:* The multiplication theorem for series requires that the two series be absolutely convergent; if this condition is not met, their product may be divergent.
Show that the series $\displaystyle\sum_0^\infty \frac{(-1)^i}{\sqrt{i+1}}$ gives an example: it is conditionally convergent, but its product with itself is divergent. (Estimate the size of the odd terms c_{2n+1} in the product.)
1 E-8.4/1 *read:* $\sum_0^\infty$
3• Ans. Q8.2/2 (p. 124) $x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \ldots = \ln(1+x)$; Abel sum is $\ln 2$ (cf. 4.2).
3• P-9-1 *add hyp.:* $a_1 > 0$, $f(a_i) = a_i$ $(i = 1, 2)$; *replace end of last line by:* for all x in I.
1• P-9-2: *replace last two sentences by:*
Show the analogous statement for $x > 0$ and a strictly *decreasing* function is false.
3• E-10.1/7a(ii) *read:* is strictly decreasing
1 E-10.3/5: *renumber:* 10.3/4
1• E-11.1/3: *add:* (Use $|\sin u| \le |u|$ for all u.)
3 E-11.1/4 *read:* exponential law, $e^{a+b} = e^a e^b$,
1• E-11.2/2: *rewrite:* Let $f(x)$ be even; prove: $\lim_{x\to 0^+} f(x) = L \;\Rightarrow\; \lim_{x\to 0} f(x) = L$.
1 E-11.3/1a: *add:* $x \ne 0$
1• E-11.3/1b: *read after semicolon:* using one of the preceding exercises.
3• E-11.3/3 *read:* b) $\lim_{x\to 0^-} \int_0^1 t^2/(1+t^4x)\,dt = 1/3$.
3 E-11.3/5 *add:* As $x \to x_0$,
1 E-11.3/6: *add:* $n > 0$
3• E-11.5/2: *rewrite:* Prove $\lim_{x\to\infty} \sin x$ does not exist by using Theorem 11.5A.
1• P-11-2: *read:* a positive number c …
3 E-12.1/3: *read:* a polynomial
3 E-12.2/1: *add at end:* Make reasonable assumptions.
3 E-12.2/3: *change:* solutions *to* zeros
1• P-12-1: *replace:* Theorems 11.3C and 11.5 *by* Theorem 11.4B
3 Ans. 12.1/4 (p. 183) *read:* $\log_2[(b-a)/e]$
2• Q-13.3/3 (p. 188): *read:* $(0, 1]$
3• E-13.1/2 *renumber as 13.2/2, and change part (b) to:*
13.2/2b Prove the function of part (a) cannot be continuous.
2• E-13.3/1: *read:* $\lim f(x) = 0$ as $x \to \pm\infty$
1 E-13.4/2: *read:* italicized property on line 2 of the …
3 E-13.5/2 *change the two R to* $\mathbf{R}$
1 P-13-5: *read:* 13.4/1
3 P-13-7 *last two lines, read:* but for the part of that argument using the compactness of $[a, b]$, substitute part (a) of 13-6 above.)

1• Ans. Q-13.3/3 (p. 195): *change to:* $\frac{1}{x}\sin(\frac{1}{x})$; as $x \to 0^+$, its swing amplitude $\to \infty$.
1• E-15.2/2b: *read:* $0 < a < 1$
1• E-15.3/2b: *read:* Prove (15) by applying the Mean-Value Theorem to

$$F(t) = f(t)\big(g(b) - g(a)\big) - g(t)\big(f(b) - f(a)\big)$$

1• P-15-2: *read:* show that between two zeros of f is a zero of g, and vice-versa
2 E-16.1/1a,b: *read:* $(0, 1]$
1 E-16.1/1c: *renumber:* 1b
1• E-16.1/3: *read:* $a \in [0, 2]$
3• E-16.2/1: *read:* converses of the implications in (8) are not true
1• P-16-1: *read:*

Prove: on an an open interval I, a geometrically convex function $f(x)$ is continuous. (Show $\lim\limits_{\Delta x \to 0^-} \Delta y/\Delta x$ exists at each point of I; deduce $\lim\limits_{\Delta x \to 0^-} \Delta y = 0$.)

3 p. 230, Ans. to Q-16.1/2 *change 9 to 0*
1• E-17.4/1c: *add:* for $-1 < x \le 0$
1• E-17.4/1d: *add:* for $0 \le x < 1$
3• E-18.2/1 *add:* Hint: cf. Question 18.2/4; use $x_i^2 - x_{i-1}^2 = (x_i + x_{i-1})(x_i - x_{i-1})$.
3 E-18.3/1 *replace n by k everywhere*
1• E-19.2/2: *read:* lower sums,
1• E-19.4/3: *change:* $\ln(6.6)$ *to* $\pi/10$
3• E-19.6/1: *(b) line 2; replace $f(x)$ by $p(x)$; if no (b), call what's in ( ) part (b)*
1• P-19-2: *make the "Prove" statement part (a), then add:*

(b) Prove the converse: if $f(x)$ on $[a, b]$ is integrable with integral $\mathcal{I}$ in the sense of the above definition, then it is integrable and its integral is $\mathcal{I}$ in the sense of definitions 18.2 and 19.2. (Not as easy as (a).)

2• E-20.3/5b: *read:* (b) In the picture, label the u-interval $[a_1, x]$ and the v-interval $[a_2, y]$. If a continuous strictly increasing elementary function $v = f(u)$ has an antiderivative that is an elementary function, the same will be true for its inverse function $u = g(v)$ (which is also continuous and strictly increasing, by Theorem 12.4).

Explain how the picture shows this.

1• E-20.5/2: *read:* give an estimate $f(n)$ for the sum C_n and prove it is correct to within 1.
1 P-20-6b: *change: 11.3B to 5.2*
3 Ans. Q-20.5/1 (p.289): *read:* 1024
3 E-21.1B - line 3: *read:* $\lim_{R\to\infty} \int_{-R}^{0}$
line -2: *read:* for $p > 1$,
1 E-21.2/2: *delete on second line:* dx
1• P-21-3: *delete hint, add hypothesis:* $\int_a^\infty f'(x)\,dx$ is absolutely convergent.
2 E-22.1/3 *read:* $u_k(x) =$
2 P-23-1 hint: *change* continuities *to* discontinuities
1• E-24.1/3: all x should be in boldface type
1• E-24.7/2: *read:* two distinct points not in S. Prove there is an x in S which…
1• P-26-2: *add:* Assume the y_i have continuous second derivatives.
1 Ans. to Q-27.2/2c, (p.396) line 2: *read:* $te^{-t} < e^{(\epsilon-1)t}$
3 E-A.4/6 *read:* Fermat's Little Theorem is the basis of the RSA encryption algorithm, widely used to guarantee website security.
3 Ans. Q-A.4/1 (p. 417) line 1: *read:* both sides are 1
3 Ans. Q-A.4/2 (p. 417) line 1: *read:* $2^n + 1$
3• E-D.2/4: *read:* By calculating y' and y'' for $x \ne 0$ and using undetermined coefficients, find a second-order linear homogeneous ODE satisfied by $y = x^4 \sin(1/x),\ y(0) = 0$.