Practice 18.01 Advanced Placement Exam – Part I   90 minutes

Same as 18.01A Exam for Admission

No books, notes, or calculators; solutions on reverse side

This exam is based on the AB syllabus, with an emphasis on differentiation. It assumes you had a year of calculus in high school, or the equivalent, and is given during R/O week at M.I.T. You do not need to (and should not) take this exam for admission to 18.01A if you meet one of the other admission criteria: getting a 4 or 5 on the AB test or as the AB subscore on the BC test, or passing the A-level or IB exam, or a comparable college calculus course (transcript and syllabus required).

However, if you are trying to advance place all of 18.01, you must take both this exam and Part II of the 18.01 AP Exam, which will be given immediately afterwards. *You must take both parts, even if you meet one of the other admission criteria for 18.01A.* The subject matter of the Part II exam is described for example on the web page

http://math.mit.edu/~apm/1801A.html

1. Find: a) $D(e^{-x^2}\tan x)$ b) $f'(1)$, if $f(u) = \cos^2(u^2+2u)$ c) $\left.\dfrac{d}{dx}\dfrac{(2x+1)^2}{x^2+1}\right|_{x=1}$

2. By implicit differentiation, find the tangent line at $(2,1)$ to the graph of $xy^3 - 2x^2y + 5x = 4$.

3. Sketch the graph of $y = 2x^3 - 3x^2 + 1$, indicating maximum and minimum points, and points of inflection.

4. 48 feet of fencing are to be used to fence off three sides of a rectangular backyard garden; the fourth side will be part of the straight house wall. What dimensions will produce the garden of biggest area?

5. Using the definition of derivative, evaluate $\displaystyle\lim_{h\to 0}\frac{(10+h)^7 - 10^7}{h}$, by relating it to a derivative.

6. Tell whether each of the following functions is continuous and/or differentiable at $x = 0$.
   a) $|x|$ b) $\dfrac{|x|}{x}$ c) $f(x) = \begin{cases} 1 - x^2, & \text{if } x < 0, \\ \cos x, & \text{if } x \ge 0. \end{cases}$

7. An open can of oil is accidentally dropped into a lake; assume the oil spreads over the surface as a circular disk of uniform thickness whose radius increases steadily at the rate 10 cm/sec. At the moment when the radius is 1 meter, the thickness of the oil slick is decreasing at the rate 4 mm/sec; how fast is it decreasing when the radius is 2 meters? (Watch the units!)

8. Find the function $y(x)$ which satisfies both the differential equation $\dfrac{dy}{dx} = e^{2x-y}$ and the condition $y(0) = 0$. (Write your answer in the form $y = f(x)$.)

9. Evaluate $\displaystyle\int_0^1 x^3(x^4+3)^{1/2}\,dx$.

10. Find the area below the graph of $y = \dfrac{\ln^3 x}{x}$, over the x-axis, and to the left of the vertical line $x = 2$.

11. Find the volume of the solid obtained when the area under the graph of $y = \dfrac{1}{2x+1}$ and over the interval $0 \le x \le a$ is rotated about the x-axis. What is the limiting volume as $a \to \infty$?

12. What estimate does the trapezoidal rule with two subdivisions (i.e., $n = 2$) give for the integral $\displaystyle\int_0^{\pi/2}\sin x\,dx$? (Give the answer to one decimal place; use $\sqrt{2} = 1.4$.)