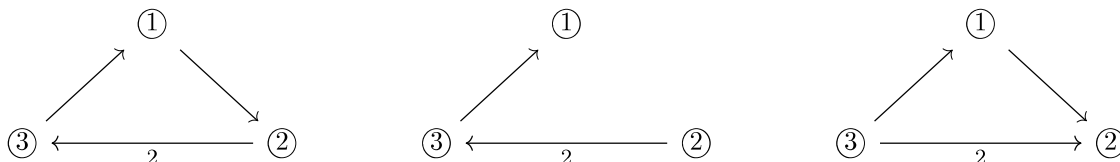




Exercise 3.1. Consider the three quivers below.



- (a) One of these quivers is of finite mutation type. Guess which one (you will prove this next, so start by taking an educated guess).
- (b) From class, we know that the quiver Q of finite mutation type must come from a surface triangulation (provided it is not one of the 11 exceptions, which it is not). Find the surface (S, M) and the corresponding triangulation such that Q is the mutable part of the corresponding quiver.

Hint: Slightly modify the annulus example from lecture. In particular, consider an annulus with 1 marked point on the inner boundary and 2 marked points on the outer boundary.

Exercise 3.2. Prove that a quiver associated to a triangulation of a surface (with no internal triangles) has arrows of only weight 0, 1, or 2.

Exercise 3.3. We've said in lecture that most cluster algebras are infinite type. Now you can show this!

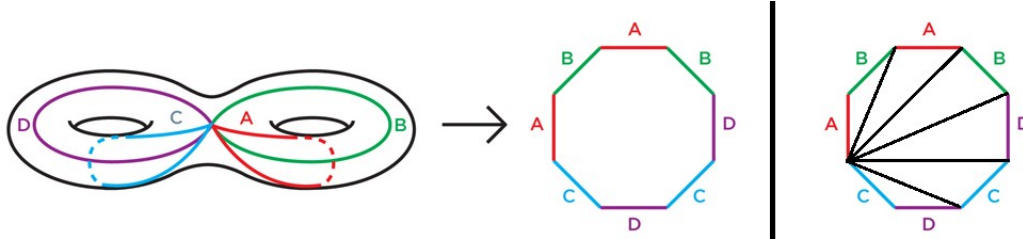
- (a) Prove that the Kronecker cluster algebra $\mathcal{A}((x_1, x_2), \bullet \xrightarrow{2} \bullet)$ is infinite type.
Hint: Specialize the initial cluster variables to 1. What can you say about the growth of the new cluster variables?
- (b) Prove that the Kronecker cluster algebra $\mathcal{A}((x_1, x_2), \bullet \xrightarrow{m} \bullet)$ is infinite type.
- (c) Show that the cluster algebra from any quiver that has an arrow of weight ≥ 2 is of infinite type.

Hint: Fix such an arrow, and the two incident cluster variables x_1 and x_2 . Specialize all other cluster variables to 1 and show that alternating mutation at 1 and 2 still yields infinitely distinct expressions.

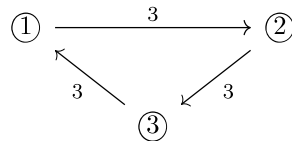
Exercise 3.4. Prove that the other two quivers from Exercise 3.1 are of infinite mutation type.

Cluster Algebras III: Additional Exercises

Exercise 3.5. By slicing the 2-torus at the four arcs below, we can view it as a gluing of an octagon⁴. Draw the rank 9 quiver $Q(T)$ given by the triangulation on the right.



Exercise 3.6. Consider the quiver Q below



If we perform the mutation sequence $1, 2, 3, 1, 2, 3, \dots$, show that the set of arrow weights that appear are always three times a Markov number (equivalently, is part of an integer solution to the equation $a^2 + b^2 + c^2 = abc$). In particular, Q is of infinite mutation type.

Exercise 3.7. Consider the marked surface (S, M) where S is a triangle and M is a single internal marked point.

- (a) Find all of triangulations of this marked surface.

Hint: Don't forget the possibility of self-folded triangles!

- (b) Draw a graph where each triangulation is a vertex, and each edge is a diagonal flip.
- (c) Describe the quiver associated to one of the triangulations with self-folded triangles.

Bonus: Give a general rule for constructing the quiver when self-folded triangles are present.

Open Problem 3.2. Two quivers are *mutation-equivalent* if they are related by a sequence of mutations.

- (a) Give an algorithm for deciding whether two quivers are mutation-equivalent.
- (b) Say anything about the complexity of this decision problem when there are no frozen vertices.

Open Problem 3.3. Give a condition for telling when a quiver Q is mutation-equivalent to a quiver Q' that has an arrow of weight exactly k .

⁴Image from Alex Bowring