Speaker: Peter May
Title: Parametrized homotopy theory
(Joint work with Jóhann Sigurðsson)

ABSTRACT

What is parametrized homotopy theory? The homotopy theory of objects (spaces or spectra, perhaps equivariant) over a given base space. What is it good for? All homotopical work that involves bundles and fibrations. Using spectra, it gives a context where one can apply the power of stable homotopy theory and still remember such basic unstable structure as fundamental groups. Is it a routine generalization of ordinary homotopy theory? No!!!. There are major conceptual and technical difficulties that have no precursors in earlier work. The force of the theory comes from base change functors that relate objects over different base spaces. Pullback base change functors have both left and right adjoints, which are used in all applicaitons. You cannot get both adjunctions on homotopy categories by just using Quillen model category theory. More serious work is needed (red flag waving). Even in the parts of the theory in which model category theory works, it doesn't work as one would expect. There is an obviously "right" naive model structure on spaces over B, but it is nonobviously useless. Similar problems arise in sheaf theoretic contexts. While our solutions to these and related problems are special to the topological context, they might serve as a guide to anyone interested in modernizing the analogous foundations in algebraic geometry. Applications? A fiberwise duality theorem allows fiberwise recognition of dualizable and invertible parametrized spectra. Application of the formal theory of duality (in symmetric monoidal categories) in the parametrized setting gives a conceptual construction and analysis of transfer maps in the nonparametrized setting. In the equivariant world, the theory gives a simple conceptual proof of a parametrized Wirthmüller isomorphism that calculates the right adjoint to base change along suitable maps in terms of a shift of the left adjoint. The Adams isomorphism relating orbit and fixed point spectra is a direct consequence. This is a fundamental result in equivariant stable homotopy theory whose original (Lewis–May) proof is impenetrable. Work in progress: this is the definitively right context in which to finally understand equivariant Poincaré duality.