Topology Seminar

Nick Kuhn

of The University of Virginia will be speaking on

Chromatic Fixed Point Theory

on October 3 at 4:30 in
MIT Room 2-131

The study of the action of a finite p-group G on a finite G-CW complex X is one of the oldest topics in algebraic topology. In the late 1930's, P. A. Smith proved that if X is mod p acyclic, then so is X^G, its subspace of fixed points. A related theorem of Ed Floyd from the early 1950's says that the dimension of the mod p homology of X will bound the dimension of the mod p homology of X^G.

The study of the Balmer spectrum of the homotopy category of G-spectra has lead to the problem of identifying "chromatic" variants of Smith's theorem, with mod p homology replaced by the Morava K-theories (at the prime p). One such chromatic Smith theorem is proved by Barthel et.al.: if G is a cyclic p-group and X is $K(n)$ acyclic, then X^G is $K(n-1)$ acyclic (and this answers questions like this for all abelian p-groups).

In work with Chris Lloyd, we have been able to show that a chromatic analogue of Floyd's theorem is true whenever a chromatic Smith theorem holds. For example, if G is a cyclic p-group, then the dimension over $K(n)_*$ of $K(n)_*(X)$ will bound the dimension over $K(n-1)_*$ of $K(n-1)_*(X^G)$.

The proof that chromatic Smith theorems imply the stronger chromatic Floyd theorems uses the representation theory of the symmetric groups.

These chromatic Floyd theorems open the door for many applications. We have been able to resolve open questions involving the Balmer spectrum for the extraspecial 2-groups. In a different direction, at the prime 2, we can show quick collapsing of the AHSS computing the Morava K-theory of some real Grassmanians: this is a non-equivariant result.

In my talk, I'll try to give an overview of some of this.

For information, write: `camkru@mit.edu`