A natural question arises when working with intersection cohomology and other stratified invariants of singular manifolds: what is the correct stable homotopy theory for these invariants to live in? But before answering that question one first has to identify the correct unstable homotopy theory of stratified spaces. The exit-path category construction of MacPherson, Treumann, and Lurie provides functor from suitably nice stratified topological spaces to “abstract stratified homotopy types” — ∞-categories with a conservative functor to a poset. Work of Ayala–Francis–Rozenblyum even shows that their conically smooth stratified topological spaces embed into the ∞-category of abstract stratified homotopy types. In this talk, we explain some of our work which goes further and produces an equivalence between the homotopy theory of all stratified topological spaces and these abstract stratified homotopy types.