 Möbius inversion is classically a procedure in number theory that inverts summation of functions over the divisors of an integer. A similar construction is possible for every locally finite poset, and is governed by a so-called Möbius function encoding the combinatorics. In 1936 Hall observed that the values of the Möbius function are Euler characteristics of intervals in the poset, suggesting a homotopy theoretic context for the inversion. In this talk we will discuss a functorial 'space-level' realization of Möbius inversion for diagrams taking values in a pointed cocomplete infinity-category. The role of the Möbius function will be played by homotopy types whose reduced Euler characteristics are the classical values, and inversion will hold up to extensions (think inclusion-exclusion but with the alternating signs replaced by even/odd spheres).

This provides a uniform perspective to many constructions in topology and algebra. Notable examples that I hope to mention include handle decompositions, Koszul resolutions, and filtrations of configuration spaces.