Welded tangles are knotted surfaces in $\mathbb{R}^4$. Bar–Natan and Dancso described a class of welded tangles which have foamed vertices—where one allows surfaces to merge and split. This classification is analogous to the planar algebras of Jones, called “circuit algebras.” In joint work with Dancso and Halacheva, we provide a correspondence between circuit algebras and a form of rigid tensor category called “wheeled props.” This is a higher-dimensional version of the well-known algebraic classification of planar algebras as certain pivotal categories. This classification allows us to connect these “welded tangled foams” to the Kashiwara-Vergne conjecture in Lie theory. In work in progress, we show that the group of homotopy automorphisms of the (rational completion of) the wheeled prop of welded foams is isomorphic to the group of symmetries KV, which acts on the solutions to the Kashiwara-Vergne conjecture. Moreover, we explain how this approach illuminates the close relationship between the group KV and the pro-unipotent Grothendieck–Teichmüller group.

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