Topology Seminar

David Spivak

of University of Oregon will be speaking on

Derived smooth manifolds

on May 4 at 4:30 in
MIT Room 2-131

Given a smooth manifold M and two submanifolds A and B, their intersection need not be a smooth manifold. By Thom's transversality theorem, one can deform A to be transverse to B and take the intersection: the result, written $A \cap B$, will be a smooth manifold. Moreover, if A and B are compact, then there is a cup product formula in cobordism, integral cohomology, etc. of the form $[A] \cup [B] = [AB]$, where $[\cdot]$ denotes the cohomology fundamental class. The problem is that AB is not unique, and there is no functorial way to choose transverse intersections for pairs of submanifolds. The goal of the theory of derived manifolds is to correct this defect. The category of derived manifolds contains the category of manifolds as a full subcategory, is closed under taking intersections of manifolds, and yet has enough structure that every compact derived manifold has a fundamental class. Even if the submanifolds A and B of M are not transverse (in which case their intersection can be arbitrarily singular), their intersection $A \times_M B$ will be a derived manifold with $[A \times_M B] = [AB]$, and thus satisfy the above cup product formula. To construct the category of derived manifolds, one imitates the constructions of schemes, but in a smooth and homotopical way. I will begin the talk by explaining this construction. Then I will give some examples and discuss some features of the category of derived manifolds. I will end by sketching the Thom-Pontrjagin argument which implies that compact derived manifolds have fundamental classes.