Simple Person's Applied Math Seminar.



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How to compute the linear response of chaotic systems

Often we want the gradients of output quantities with respect to input parameters (like control/design variables), for parameter estimation, uncertainty quantification, inside an optimization algorithm and so on. Tangent linear models, adjoint methods (or backpropagation), numerical finite-differencing, automatic differentiation etc have been developed to compute these gradients and used extensively in nonchaotic systems. In the chaotic case, we typically want the gradients of infinite time averages or equivalently if the system is ergodic with respect to a probability measure, gradients of expectations with respect to that measure. The aforementioned linear perturbation methods and naive improvisations (such as ensemble sensitivity methods) do not produce meaningful information when the underlying nonlinear dynamics is chaotic. The reason is that infinitesimal perturbations to a chaotic state are almost surely unstable under time evolution. In this talk, a new method called space-split sensitivity or S3, will be introduced to differentiate ergodic probability measures (which typically do not have densities) in chaotic systems, at a convergence rate of a typical Monte Carlo simulation. The S3 algorithm uses Oseledets splitting and linear response theory for uniformly hyperbolic attractors. These keys ideas will be discussed with the help of fun pictures of attractors scattered across the talk.

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