Every animal cell is filled with a cytoskeleton, a dynamic gel made of inextensible fibers, such as microtubules, actin fibers, and intermediate filaments, all suspended in a viscous fluid. Numerical simulation of this gel is challenging because the fiber aspect ratios can be as large as $10^4$ and the hydrodynamic interactions among the filaments are long ranged. We describe a new method for rapidly computing the dynamics of inextensible slender filaments in periodically-sheared Stokes flow. The dynamics of the filaments are governed by a nonlocal slender body theory, which we reformulate in terms of the Rotne-Prager-Yamakawa hydrodynamic tensor. To enforce inextensibility, we parameterize the space of inextensible fiber motions and strictly confine the dynamics to the manifold of inextensible configurations, and apply the constraint of no virtual work to the Lagrange multipliers for the tensile force densities on the filaments. I will compare two spatial discretizations: a spectral discretization based on Chebyshev polynomials, and a more standard direct discretization using a chain of blobs (beads), and briefly discuss fast and accurate implementation of quadratures/sums involving hydrodynamic kernels. For dynamics, we develop a second-order semi-implicit temporal integrator which requires at most a few evaluations of nonlocal hydrodynamics and a few block diagonal linear solves per time step. We apply our formulation to a permanently cross-linked actin mesh in a background oscillatory shear flow, and find that far-field hydrodynamics increases the viscous modulus by as much as 25%, even for semi-dilute fiber suspensions. Future extensions to include fiber twist and thermal fluctuations will be discussed as time permits.