THE TWO-HYPERPLANE CONJECTURE

The main theme of this talk is that least energy solutions of variational problems, such as area minimizing surfaces, free boundaries and eigenfunctions, should be as simple as possible, but no simpler.

We'll start by discussing the *Hot Spots Conjecture* of Jeff Rauch, which says that (generically) the hottest point of a perfectly insulated room tends to a wall (or to the floor or ceiling) as time tends to infinity. This poetic description is a disguise for the precise formulation, which says that the maximum (and minimum) of a least energy non-constant Neumann eigenfunction is achieved on the boundary. For me the essential aspect of the prediction is that the level surfaces of the eigenfunction are as simple as possible. This question is so delicate that it has not even been resolved for level curves of eigenfunctions of triangles in the plane, but I believe the conjecture is true for convex regions in all dimensions.

I will make a leap from Rauch's conjecture to an array of "easier" questions concerning the shape of solutions to more general linear and semilinear elliptic partial differential equations. I will explain how these are directly related to questions about area-minimizing surfaces and free boundaries. Then I will introduce a conjecture that I will call the *Two Hyperplane Conjecture*, in the spirit of a famous conjecture of Kannan, Lovász and Simonovits in theoretical computer science known as the *Hyperplane Conjecture*. Their conjecture turns out to be essentially equivalent to a long list of famous conjectures in high dimensional convex geometry. Both the hyperplane and two-hyperplane conjecture say that least perimeter hypersurfaces resemble hyperplanes, so that, in some sense, they are as simple as possible. But unlike the hyperplane conjecture, the two-hyperplane conjecture has significance even in low dimensions.