Massachusetts Institute of Technology
Department of Mathematics

LUNCH SEMINAR FOR GRADUATE STUDENTS

Monday, October 15, 2012
12:00 - 1:00 PM   Room 2-143

Larry Guth
(MIT)

"The waist inequality in geometry and combinatorics"

Abstract

Abstract: Consider a continuous map from the unit n-sphere to $\mathbb{R}^q$. What can we say about the level sets of the map? Is it possible they are all small? For example, if we take a linear map, then each level set will be an $(n-q)$-sphere, and the largest level set will be an $(n-q)$-equator. In fact, this linear map is optimal: for any continuous map, one of the level sets has $(n-q)$-dimensional volume at least as large as an equator. This inequality is called the waist inequality. For $q = 1$ it is quite old, but for q at least 2 it comes from the 60's or even later, and it is very difficult to prove. It's even difficult to prove that every continuous map has a level set of $(n-q)$-volume at least some tiny positive constant epsilon(q, n).

We will discuss this inequality and its role in geometry. We discuss the difficulty of proving the result and give a sketch of the proof. We can replace the unit n-sphere by other Riemannian manifolds, and we explore what happens.

Also, we discuss a theorem in combinatorics called point selection, first proven by Barany in the 1980's. Recently, Gromov observed that this discrete combinatorial theorem is analogous to the waist inequality, and he adapted one proof of the waist inequality to give a new proof of this theorem in combinatorics.

Followed by pizza in room 2-290