

LUNCH SEMINAR FOR GRADUATE STUDENTS

Monday, April 11, 2011
12:00 - 1:00 PM Room 2-147

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“How to construct topological invariants via decompositions and the symplectic category”

Abstract

A Lagrangian correspondence is a Lagrangian submanifold in the product of two symplectic manifolds. This generalizes the notion of a symplectomorphism and was introduced by Weinstein in an attempt to build a symplectic category that has morphisms between any pair of symplectic manifolds (not just symplectomorphic pairs). In joint work with Chris Woodward we define such a category, in which all Lagrangian correspondences are composable morphisms.

We extend it to a 2-category by extending Floer homology to cyclic sequences of Lagrangian correspondences. This is based on counts of ‘holomorphic quilts’ — a collection of holomorphic curves in different manifolds with ‘seam values’ in the Lagrangian correspondences. A fundamental isomorphism of Floer homologies ensures that our constructions are compatible with the geometric composition of Lagrangian correspondences.

This provides a general prescription for constructing topological invariants. We consider e.g. 3-manifolds or links as morphisms (cobordisms or tangles) in a topological category. In order to obtain a topological invariant from our generalized Floer homology, it suffices to

- decompose morphisms into simple morphisms (e.g. by cutting between critical levels of a Morse function)
- associate to the objects and simple morphisms smooth symplectic manifolds and Lagrangian correspondences between them (e.g. using moduli spaces of bundles or representations)
- check that the moves between different decompositions are associated to (good) geometric composition of Lagrangian correspondences

Followed by pizza in room 2-290