September 11, 2019: David Vogan (MIT), *Centralizers of nilpotent elements.*

Report on work of Jeffrey Adams and Annegret Paul.

Suppose G is a complex connected reductive algebraic group, and $X \in \mathfrak{g}$ is a nilpotent element. For many purposes in representation theory it is important to understand the algebraic group

$$G^X = \{g \in G \mid \mathrm{Ad}(g)(X) = X\}.$$

Jacobson, Morozov, and Kostant proved that there is an algebraic group homomorphism

$$\phi\colon SL(2) \to G, \qquad d\phi(\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}) = X,$$

defined uniquely up to conjugation by G^X. Then the (possibly disconnected) reductive group

$$G^\phi \subset G^X$$

is a Levi factor; so G^X is the semidirect product of G^ϕ and the unipotent radical.

There has not been much work on understanding the unipotent radical (although this is certainly very important). The finite group of connected components

$$A(X) = G^X/G^X_0 = G^\phi/G^\phi_0$$

has been tabulated in all cases, and so has the Lie type (number and ranks of type A factors ...) of G^ϕ_0.

The full disconnected reductive group G^ϕ is more or less completely computed in Lusztig's paper "Unipotent almost characters of simple p-adic groups, II," *Transform. Groups* **19** (2014), no. 2, 527–547, using earlier work of Liebeck and Seitz among others.

I will describe work done by Adams and Paul toward computing $A(X)$ and the root datum of G^ϕ_0 in the `atlas` software, and some conjectures about structure theory they would like to use to describe the disconnected group G^ϕ completely.