## April 29: Lucas Mason-Brown Unipotent representations of real reductive groups.

Let G be a real reductive group and let  $\widehat{G}$  be the set of irreducible unitary representations of G. The determination of  $\widehat{G}$  (for arbitrary G) is one of the fundamental unsolved problems in representation theory. In the early 1980s, Arthur introduced a finite set  $\operatorname{Unip}(G)$  of (conjecturally unitary) irreducible representations of G called *unipotent representations*. In a certain sense, these representations form the building blocks of  $\widehat{G}$ . Hence, the determination of  $\widehat{G}$  requires as a crucial ingredient the determination of  $\operatorname{Unip}(G)$ . In this thesis, we prove three results on unipotent representations. First, we study unipotent representations by restriction to  $K \subset G$ , a maximal compact subgroup. We deduce a formula for this restriction in a wide range of cases, proving (in these cases) a long-standing conjecture of Vogan. Next, we study the unipotent representations attached to induced nilpotent orbits. We find that  $\operatorname{Unip}(G)$  is generated by an even smaller set  $\operatorname{Unip}'(G)$  consisting of representations attached to rigid nilpotent orbits. Finally, we study the unipotent representations attached to the principal nilpotent orbit. We provide a complete classification of such representations, including a formula for their K-types.