February 6: David Vogan (MIT), "Hodge filtrations on Harish-Chandra modules (after Schmid-Vilonen and Adams)."

Suppose $G_{\mathbb{R}}$ is a real reductive Lie group and $K_{\mathbb{R}}$ is a maximal compact subgroup. Write $\mathfrak{g}$ for the complexified Lie algebra of $G_{\mathbb{R}}$ and K for the complexification of $K_{\mathbb{R}}$. Harish-Chandra showed that many (analytic) problems in the representation theory of $G_{\mathbb{R}}$ could be reformulated as (algebraic) problems about $(\mathfrak{g}, K)$ modules.

A fundamental technique for approaching such problems is the use of a *good filtration* of a $(\mathfrak{g}, K)$ module. A fundamental limitation on this technique is that good filtrations are far from unique. Around 2011 Schmid and Vilonen used very deep ideas from Saito's theory of mixed Hodge modules to introduce a *canonical* good filtration (the *Hodge filtration*) on at least some $(\mathfrak{g}, K)$ modules, including all the irreducible ones. They used their filtrations to formulate a conjectural description of the unitary dual of $G_{\mathbb{R}}$.

I will sketch the construction of Schmid and Vilonen, and explain what it could mean to "compute" the Hodge filtration. Then I will outline work of Jeff Adams on software to carry out this computation. The software works, but we are lacking some theoretical ideas to prove that what it computes is in fact the Hodge filtration. I will try to explain these gaps. For me the most appealing problem is to find a direct representation-theoretic description of the Hodge filtration, bypassing Saito's work.