April 4: David Vogan (MIT), *Nilpotent orbits and Weyl group representations.*

The theory of the symmetric group may be considered to originate in the work of Cauchy around 1845–46. Frobenius in 1900 found that the irreducible representations of S_n may be parametrized like conjugacy classes of nilpotent $n \times n$ matrices, by partitions of n. Because the symmetric group and nilpotent matrices are closely connected to $GL(n)$, this immediately suggests the possibility of a direct relationship between the two parametrizations.

In 1955 J. A. Green found such a relationship, seeing the character table for S_n as a degeneration (at $q = 0$) of part of his character table for $GL(n, \mathbb{F}_q)$. Green's ideas were pushed much further by Springer (geometrically) and by Lusztig (representation-theoretically). One of the endpoints is a 2009 paper of Lusztig relating the unipotent classes in a reductive group over an algebraically closed field to representations of the corresponding Weyl group.

I will outline some of this work, with emphasis on interpretations related to infinite-dimensional representations of real reductive groups, and I will try to explain some open problems (such as explaining the apparent period of five-ninths of a century for fundamental advances in this direction).