March 1: Seth Shelley-Abrahamson (MIT), Counting irreducible representations of rational Cherednik algebras of given support.

Given a finite complex reflection group W with reflection representation \mathfrak{h} , one can consider the associated rational Cherednik algebras $H_c(W, \mathfrak{h})$ and their representation categories $\mathcal{O}_c(W, \mathfrak{h})$. The irreducible representations in $\mathcal{O}_c(W, \mathfrak{h})$ are in natural bijection with the irreducible representations of W, and each representation M in $\mathcal{O}_c(W, \mathfrak{h})$ has an associated support $\operatorname{Supp}(M)$, a closed subvariety of \mathfrak{h} . There is then a natural question: given a closed subvariety $X \subset \mathfrak{h}$, how many irreducible representations $L_c(\lambda)$ have support precisely equal to X?

In this talk I will explain how to answer this question when W is an arbitrary finite Coxeter group. In the case of full support, i.e. $X = \mathfrak{h}$, the answer is essentially given by the KZ functor. In particular, KZ establishes a natural bijection between the irreducible representations in $\mathcal{O}_c(W,\mathfrak{h})$ of full support and the irreducible representations of the Hecke algebra $H_q(W)$, where the parameter q depends on the parameter c in an exponential manner. To handle the proper support case, I will define a certain filtration of $\mathcal{O}_c(W,\mathfrak{h})$ by Serre subcategories that is a refinement of the filtration by supports. The subquotients of this filtration are naturally labeled by W-orbits of "cuspidal pairs" (W', L), where $W' \subset W$ is a parabolic subgroup and L is a finite-dimensional irreducible representation of $H_c(W', \mathfrak{h}/\mathfrak{h}^{W'})$; this should be seen as an analogue for rational Cherednik algebras of the Harish-Chandra series appearing in the representation theory of finite groups of Lie type. Generalizing the KZ functor, I will explain the construction of a monodromy functor KZ_L that will identify the subquotient labeled by (W', L) with the category of finite-dimensional representations over a generalized Hecke algebra of precisely the form considered by Howlett and Lehrer in the setting of finite groups of Lie type. A 2-cocycle appears in this setting as well, and by casework it can be verified that it is always trivial. Finally, I will explain how the parameters of the generalized Hecke algebras can be efficiently computed. This is joint work with I. Losev.