October 9, 2013: David Vogan (MIT), Leading terms of characters (continued).

Joint work with Pavle Pandzić and Salah Mehdi.

Suppose $\pi(\lambda_a)$ is an irreducible representation of a real reductive group G having regular infinitesimal character $\lambda_a \in \mathfrak{h}_a^*$. The Jantzen-Schmid-Zuckerman theory of "coherent continuation" provides a family of virtual representations $\pi(\lambda_a + \mu_a)$ for any μ_a in the lattice of algebraic characters. Harish-Chandra's distribution character $\Theta_{\pi(\lambda_a + \mu)}$ is a function on the set of regular semisimple elements of G.

In a talk September 4, I explained that the numerator of this character has (on appropriate components of the regular set in each Cartan subgroup H of G) a formula shaped like

$$\sum_{w \in W} a_w \exp(\langle x, w(\lambda + \mu) \rangle) \qquad (x \in \mathfrak{h}_0).$$

Here the coefficients $a_w \in \mathbb{Z}$ depend on π .

The space of all functions of this form (still for a fixed component of a fixed H) carries two commuting actions of W: on the x variable ("left") and on the λ variable ("right"). The resulting representation of $W \times W$ is clearly just the regular representation of W.

I will discuss what results of Joseph and Lusztig have to say about the right action of W on the character; what results of Springer have to say about the left action; and how these ideas fit together. With a little luck, I will even mention what was supposed to be the topic September 4: the relationship of these formulas to index of Dirac operators.