February 19, 2014: David Vogan (MIT), *W-graphs and Harish-Chandra modules*

A W-graph is a very simple combinatorial structure (created by Kazhdan and Lusztig in 1979) encoding a representation of the Hecke algebra of a Coxeter group (W, S). (I will recall their definition.) Kazhdan and Lusztig showed how to create a large family of interesting W-graphs, each having vertex set a subset of W, which turned out also to encode interesting information about highest weight representations, including formulas for their characters. Subsequent work found W-graphs related to Harish-Chandra modules for real groups.

I'll discuss two problems that have attracted a lot of attention, but are far from completely understood: how to extract additional representation-theoretic information (for example, associated varieties) from W-graphs; and how to understand "all" W-graphs in a combinatorial way. Most of this work has centered on Kazhdan and Lusztig's W-graphs for highest weight representations, but I'll look at some interesting examples from Harish-Chandra modules.

My original goal in giving this talk (not achieved!) was to understand a recent example of Williamson of a new kind of reducibility in the characteristic cycle of an irreducible highest weight module for $GL(12)$. I'll at least explain his example.