The Heisenberg–Weil Representation and Fast Wireless Communication

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Abstract: The vector space $\mathcal{H} = \mathbb{C}(\mathbb{Z}/N)$ of complex valued functions on $\mathbb{Z}/N = \{0, ..., N-1\}$ is called in electrical engineering the Hilbert space of digital signals. An important task for doing efficient wireless communication is the algorithmic construction of 'good signals' in $\mathcal{H}$. In my lecture I will explain how to use the Heisenberg–Weil representation, i.e., the natural symmetries of the space $\mathcal{H}$, to construct 'good signals'. These signals will enable us to present a new algorithm, called the "Flag Method", that suggests a solution to the following problem: You transmit your 'good signal' $S(t) \in \mathcal{H}$, to the antenna; the antenna receives the signal $R(t) \in \mathcal{H}$ which has the form

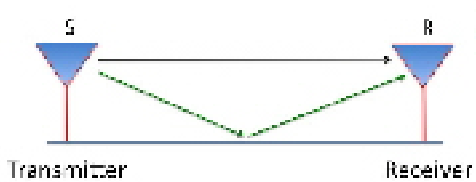
$$R(t) = e^{\frac{2\pi i}{N}\omega_0 t} \cdot S(t + \tau_0) + W(t),$$

where $W \in \mathcal{H}$ is a random signal, $\tau_0, \omega_0 \in \mathbb{Z}/N$ encodes the time took the signal to arrive to the antenna, and your radial velocity with respect to the antenna, respectively.

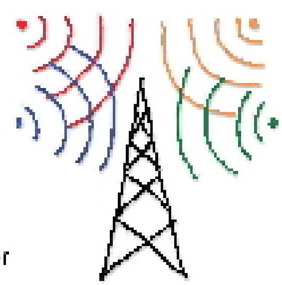
Problem (Channel Estimation) Assume you have R and S. Extract τ_0, ω_0.

In my lecture I first introduce the classical matched filter algorithm that suggests the 'traditional' way (using fast Fourier transform) to solve the channel estimation problem in order of $N^2 \log(N)$ arithmetical operations. Then I will explain how the flag method solves this problem in a much faster way of order of $N \log(N)$ operations (in certain applications $N >> 1,000$).

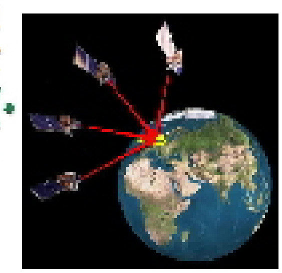
Finally, I will explain applications of our method to mobile communication, and global positioning system (GPS).



Channel Estimation



Mobile Communication



GPS

This is a joint work with A. Fish (Math, Madison), R. Hadani (Math, Austin), A. Sayeed (EE, Madison), and O. Schwartz (EECS, Berkeley).

I will assume knowledge of elementary linear algebra.