March 14: David Vogan (MIT), "What the local Langlands correspondence can do for you."

Langlands' program (mostly conjectural) says that automorphic representations π on a reductive group G (defined over a number field k) should be indexed by arithmetic parameters, related to the Galois group of k and to the complex dual group ${}^\vee G$ of G. It's a theorem that π is a tensor product over places v of k of representations π_v of $G(k_V)$. The local Langlands program (proved for GL_n and for all groups over $\mathbb{R}$) says that representations π_v should be indexed by local arithmetic parameters, related to the Galois group of k_v and to ${}^\vee G$. (Then there should be a big commutative diagram ...)

Over a p-adic field k_v, Langlands' precise formulation of these statements was improved by Deligne and by Lusztig to a rather complete (conjectural) description of representation theory for $G(k_v)$ in terms of (complex algebraic) geometry on ${}^\vee G$. Over $\mathbb{R}$, Langlands precise and proven formulation was (I claim) not the right one. This talk is an advertisement for a different formulation, due (long long ago) to Adams, Barbasch, and me. Main theorem/example will be a description of "blocks" of representations of $G(\mathbb{R})$ (equivalence classes under the relation "has a non-trivial extension with") in terms of (roughly speaking) real forms of ${}^\vee G$.

The number-theoretic content of this is that the definition of the real Weil group is not the right one. I'll explain the classical definition and the replacement.