September 10: David Vogan (MIT), "Finite maximal tori (after Patera, Zassenhaus, Bahturin, and a host of others)."

Let $\mathfrak{g}$ be a complex semisimple Lie algebra, and G the identity component of its automorphism group. A "finite maximal torus" for G is a maximal abelian subgroup A of G that is also finite. Such subgroups have been classified in many cases (including all the classical simple Lie algebras). Each gives rise to a grading

$$\mathfrak{g} = \sum_{\xi \in \widehat{A}} \mathfrak{g}_\xi$$

according to the characters by which A acts on $\mathfrak{g}$. The *roots of A in $\mathfrak{g}$, $R(\mathfrak{g}, A)$* are the characters ξ for which $\mathfrak{g}_\xi \neq 0$.

This decomposition behaves formally much like the usual root decomposition, but the finiteness of A makes two great differences: the trivial eigenspace $\mathfrak{g}_1$ is zero; and each space $\mathfrak{g}_\xi$ is the Lie algebra of a torus (rather than a unipotent subgroup).

Just as combinatorial study of the classical root system reveals interesting subgroups of G, so a study of the finite geometries $R(\mathfrak{g}, A)$ reveals (quite different) subgroups of G.

I will discuss the classification of finite maximal tori in the classical case, and some interesting examples for the exceptional groups.