April 22: Bertram Kostant (MIT), "Exotic finite subgroups of $E_{8}$ and T. Springer's regular elements of the Weyl group." (Joint work with N. Wallach). Let $\mathfrak{g}$ be a complex
simple Lie algebra and let $G$ be the adjoint group of $\mathfrak{g}$. Let $h$ be the Coxeter number of $\mathfrak{g}$. Some time ago I conjectured that if $q=2 h+1$ is a prime power then the finite simple group $L_{2}(q)$ embeds into $G$. With the help of computers, in a number of the cases, this has been shown to be true. The most sophisticated case is when $G=E_{8}$. Here $q=61$. This embedding was first computer established by Cohen-Griess and later without computer by Serre. Griess-Ryba also later (computer) proved that $L_{2}(49)$ and $L_{2}(41)$ embed into $E_{8}$.
Write the three prime powers $61,49,41$ as $q_{k}$ where $k=30,24,20$ so that $q_{k}=2 k+1$. In a 1959 paper I related, for any simple $\mathfrak{g}$, the Coxeter element with the principal nilpotent element in $\mathfrak{g}$. Tonny Springer, in a 1974 paper, extending my result in the special case of $E_{8}$, established a similar connection, between three nilpotent elements, $e_{k} \in \mathfrak{g}$, and three (regular) elements $\sigma_{k}$ of the Weyl group. The order of $\sigma_{k}$ is $k$. Using some beautiful properties of $\sigma_{k}$ the main result in our presentation this week is the establishent of a clear cut connection between Springer's result, on one hand, with the Griess-Ryba embedding $L_{2}\left(q_{k}\right)$ in $E_{8}$ on the other.

