

September 13: David Vogan (MIT), “How many jellybeans are in that jar?”

Jeff Adams’ research group “Atlas of Lie groups and representations” is getting ready to do a big computation: of the Kazhdan-Lusztig polynomials attached to the split real group of type E_8 . The software to do this was written by Fokko du Cloux, and there is a supercomputer being built at the University of Oklahoma which will try to do the computation, under the supervision of Birne Binengar. I will explain a number of mathematical questions related to this computation. The most fundamental is the one of the seminar title: how many distinct Kazhdan-Lusztig polynomials (for E_8) are there? The best upper bound we now know is 7 billion. If that’s correct, then the calculation needs about a terabyte of RAM, and we can’t do it. My best (optimistic!) guess is that there are about half a billion. If that’s correct, then the calculation needs about 150G of RAM, and the Oklahoma supercomputer will do it as soon as they figure out how to turn on the air conditioning.

As a teaser, here is one of the 10,147,580 distinct Kazhdan-Lusztig polynomials for the non-split real form of E_8 polynomials. (For this group, the *a priori* upper bound on the number of polynomials was 130 million. I picked this polynomial because the coefficient 2545 is the largest one that occurs.)

$$3q^{13} + 30q^{12} + 190q^{11} + 682q^{10} + 1547q^9 + 2364q^8 \\ + 2545q^7 + 2031q^6 + 1237q^5 + 585q^4 + 216q^3 + 60q^2 + 11q + 1$$