December 1: Bertram Kostant (MIT), "The Coxeter element and the branching law for finite subgroups of $SU(2)$."

Let Γ be a finite subgroup of $SU(2)$ and let $\widetilde{\Gamma} = \{\gamma_i \mid i \in J\}$ be the unitary dual of Γ. The unitary dual of $SU(2)$ may be written $\{\pi_n \mid n \in \mathbb{Z}_+\}$ where $dim\, \pi_n = n+1$. For $n \in \mathbb{Z}_+$ and $j \in J$ let $m_{n,j}$ be the multiplicity of γ_j in $\pi_n|\Gamma$. Then we collect this branching data in the formal power series,

$$m(t)_j = \sum_{n=0}^{\infty} m_{n,j}\, t^n$$

One shows that there exists a polynomial $z(t)_j$ and known positive integers a, b (independent of j) such that

$$m(t)_j = \frac{z(t)_j}{(1-t^a)(1-t^b)}$$

The problem is the determination of the polynomial $z(t)_j$. If $o \in J$ is such that γ_o is the trivial representation then it is classical that $z(t)_o = 1 + t^h$ for a known integer h. The problem reduces to case where γ_j is nontrivial.

The McKay correspondence associates to Γ a complex simple Lie algebra $\mathfrak{g}$ of type A-D-E. We explicitly determine $z(t)_j$ for $j \in J - \{o\}$ using the orbits of a Coxeter element on the set of roots of $\mathfrak{g}$.

Mysteriously the polynomial $z(t)_j$ has arisen in a completely different context in some papers of Lusztig. Also Rossmann has recently shown that the polynomial $z(t)_j$ yields the character of γ_j.